REPRESENTATION THEORY OF FRAMISATIONS OF KNOT ALGEBRAS

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The Iwahori-Hecke algebra of type A

q indeterminate or $q \in \mathbb{C}^*$, $R = \mathbb{C}(q)$, $n \in \mathbb{N}$

$$\mathcal{H}_n(q) = \left\langle g_1, \ldots, g_{n-1} \;\middle|\; \begin{array}{ll} g_i g_{i+1} g_i = g_{i+1} g_i g_{i+1} & \\ g_i g_j = g_j g_i & , |i-j|>1 \\ g_i^2 = (q-1) g_i + q & \end{array} \right\rangle$$

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- Ocneanu trace on $\mathcal{H}_n(q) \xrightarrow[\text{re-scaling}]{\text{normalisation}}$ HOMFLYPT polynomial

The Yokonuma-Hecke algebra of type A

q indeterminate or $q \in \mathbb{C}^*$, $R = \mathbb{C}(q)$, $d, n \in \mathbb{N}$

$$Y_{d,n}(q) = \left\langle \begin{array}{c} g_1, \ldots, g_{n-1} \\ t_1, \ldots, t_n \end{array} \;\middle|\; \begin{array}{rcll} g_i g_{i+1} g_i & = & g_{i+1} g_i g_{i+1} & \\ g_i g_j & = & g_j g_i & , \ |i-j|>1 \\ t_i t_j & = & t_j t_i & \\ t_j^d & = & 1 & \\ t_j g_i & = & g_i t_{s_i(j)} & \\ g_i^2 & = & (q-1) e_i g_i + q & \end{array} \right\rangle$$

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- $Y_{d,n}(1) \cong \mathbb{C}[G(d,1,n)]$ where $G(d,1,n) = (\mathbb{Z}/d\mathbb{Z})^n \rtimes \mathfrak{S}_n$

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$$\Bigg\downarrow \begin{array}{l} \text{normalisation} \\ \text{re-scaling} \end{array} \Bigg\} \text{ E-system}$$

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$\not\equiv$

HOMFLYPT [CJKL]

Representation theory of $G = GL_n(\mathbb{F}_q)$

$$\mathcal{H}_n(q) \cong \mathrm{End}_G(\mathrm{Ind}_B^G \mathbb{1}) \qquad B = \begin{pmatrix} * & & * \\ & \ddots & \\ 0 & & * \end{pmatrix}$$

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$$Y_{d,n}(q) \cong \mathrm{End}_G\left(\mathrm{Ind}_U^G \mathbb{1}\right) \qquad U = \begin{pmatrix} 1 & \cdots & * \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$$d = q - 1$$

Representation theory of $Y_{d,n}(q)$

- Thiem 2005 : Unipotent Hecke algebras

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$\mathrm{Irr}(Y_{d,n}(q)) \longleftrightarrow \mathrm{Irr}(G(d,1,n)) \longleftrightarrow \{$ d-partitions of size $n\}$

A d-partition of size n is a family of d-partitions $\lambda = (\lambda^{(1)}, \ldots, \lambda^{(d)})$ such that $\sum(\text{size of } \lambda^{(i)}) = n$.

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Let $\vartheta : q \longmapsto \eta \in \mathbb{C}^*$

$Y_{d,n}(\eta)$ semisimple $\iff$ $\vartheta(s_\lambda) \neq 0 \ \forall \lambda$ $\iff$ η is not a root of unity of order $\leqslant n$ $(\eta \neq 1)$

$\mathbb{C}[G(d,1,n)]$

Yokonuma-Hecke algebra	Ariki-Koike algebra
$Y_{d,n}(q)$	$H(d,n)$
• Semi-direct product	• Quadratic relation
• Braid group of type A	• Braid group of type B

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$H(d,n)$

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Markov trace on Ariki-Koike algebra [Lambropoulou, Geck - L.]

$\downarrow$

Invariant for classical knots in the solid torus

The cyclotomic and the affine Yokonuma-Hecke algebra

$$
\begin{tikzcd}
Y_{d,n}(q) & & H(m,n) & & \\
& Y(d,m,n) \arrow[ul, two heads] \arrow[ur, two heads] & m < \infty & & \text{Cyclotomic} \\
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Markov trace on $Y(d,m,n)$ [C.-Poulain d'Andecy]

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\text{Affine Hecke} \arrow[rr, "\text{Replace Iwahori sbgp}", "\text{by pro-}p\text{-Iwahori sbgp}"'] & & \text{Affine Yokonuma-Hecke}
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$\wr\wr$ [C.-Sécherre]

pro-p-Iwahori-Hecke

[Vignéras]

Temperley-Lieb algebra

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$$TL_n(q) = \mathcal{H}_n(q) / \langle 1 + g_1 + g_2 + g_1 g_2 + g_2 g_1 + g_1 g_2 g_1 \rangle$$

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Yokonuma-Temperley-Lieb algebra

[Goundaroulis-Juyumaya-Kontogeorgis-Lambropoulou]

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$$YTL_{d,n}(q) = Y_{d,n}(q) / \langle 1 + g_1 + g_2 + g_1g_2 + g_2g_1 + g_1g_2g_1 \rangle$$

$$\mathrm{Irr}(Y_{d,n}(q)) \longleftrightarrow \{ d\text{-partitions of } n \} \longleftrightarrow \mathrm{Irr}(G(d,1,n))$$

$$\mathrm{Irr}(YTL_n(q)) \longleftrightarrow \{ \lambda \text{ s.t. } \square\square\square \nmid \mathrm{Res}^{G(d,1,n)}_{G_3}(V^\lambda) \}$$

[C.-Pouchin]

$$\longleftrightarrow \{ \lambda \text{ s.t. } \sum_{i=1}^{d} \lambda_1^{(i)} \leqslant 2 \}$$

$$\dim_R YTL_{d,n}(q) = \frac{d(nd-n+d+1)}{2} C_n - d(d-1)$$

$B_{YTL} = \{ t_1^{a_1} \dots t_n^{a_n} g_w \mid w \in A,$ a_j depend on w $\}$

Framisation of the Temperley-Lieb algebra

[Goundaroulis - Juyumaya - Kontogeorgis - Lambropoulou]

$n \geqslant 3$

$$FTL_{d,n}(q) = Y_{d,n}(q) \big/ \langle e_1 e_2 (1 + g_1 + g_2 + g_1 g_2 + g_2 g_1 + g_1 g_2 g_1) \rangle$$

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$$\dim_{\mathbb{R}} FTL_{d,n}(q) = \sum_{k_1 + \cdots + k_d = n} \left(\frac{n!}{k_1! \cdots k_d!} \right)^2 C_{k_1} \cdots C_{k_d}$$

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$B_{FTL} = ?$