# Rouquier blocks of the cyclotomic Hecke algebras of complex reflection groups

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February 22, 2008

# Preliminaries

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**Definition**

The blocks of $A$ are the primitive idempotents of $ZA$.

Since $KA$ is semisimple, we have a bijection

$$\begin{array}{ccc} \mathrm{Irr}(KA) & \leftrightarrow & \mathrm{Bl}(KA) \\ \chi & \leftrightarrow & e_\chi \end{array}$$

## Proposition

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In particular, the set $\{e_B\}_{B \in \mathrm{Bl}(A)}$ is the set of all the blocks of $A$.

If $\chi \in B$ for $B \in \mathrm{Bl}(A)$, we say that "$\chi$ belongs to the block $e_B$".

# Symmetric algebras

## Definition

We say that a linear map $t : A \to \mathcal{O}$ is a symmetrizing form on $A$ or that $A$ is a symmetric algebra if

- $t$ is a trace function, *i.e.*, $t(ab) = t(ba)$ for all $a, b \in A$.
- The morphism

$$\hat{t} : A \to \mathrm{Hom}_{\mathcal{O}}(A, \mathcal{O}),\ \ a \mapsto (x \mapsto \hat{t}(a)(x) := t(ax))$$

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**Example:**

*If $\mathcal{O} = \mathbb{Z}$ and $A = \mathbb{Z}[G]$ ($G$ a finite group), we can define the following symmetrizing form ("canonical") on $A$*

$$t : \mathbb{Z}[G] \to \mathbb{Z},\ \sum_{g \in G} a_g g \mapsto a_1.$$

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**Example:**
*If $\mathcal{O} = \mathbb{Z}$, $A = \mathbb{Z}[G]$ ($G$ a finite group) and $t$ is the canonical form on $A$, we have*
$$s_\chi = \frac{|G|}{\chi(1)}.$$

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- We choose a set of indeterminates

$$\mathbf{u} = (u_{s,j})_{s,\, 0 \leq j \leq \mathbf{o}(s)-1}$$

  where $s$ runs over the set of generators of $W$ and $\mathbf{o}(s)$ denotes the order of $s$ (if $s$ and $t$ are conjugate in $W$, then $u_{s,j} = u_{t,j}$ for all $j$).

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  - $u_j \mapsto (-1)^j$ $(j=0,1)$, $\mathcal{H}(G_2) \mapsto \mathbb{Z}[G_2]$.

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  - $u_j \mapsto \zeta_3^j$ $(j=0,1,2)$, $\mathcal{H}(G_4) \mapsto \mathbb{Z}_K[G_4]$.

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  - $t$ satisfies some other condition.

## Theorem (Malle)

Let $\mathbf{v} = (v_{s,j})_{s,j}$ be a set of indeterminates such that, for all $s, j$, we have

$$v_{s,j}^{|\mu(K)|} := \zeta_{\mathbf{o}(s)}^{-j} u_{s,j}.$$

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By "Tits' deformation theorem", we know that the specialization $v_{s,j} \mapsto 1$ induces a bijection

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# Generic Schur elements

**Theorem (C.)**

The Schur element $s_\chi(\mathbf{v})$ associated with the irreducible character $\chi_{\mathbf{v}}$ of $K(\mathbf{v})\mathcal{H}$ is an element of $\mathbb{Z}_K[\mathbf{v},\mathbf{v}^{-1}]$ whose irreducible factors (in $K[\mathbf{v},\mathbf{v}^{-1}]$) are of the form

$$\Psi(M)$$

where

- $\Psi$ is a $K$-cyclotomic polynomial in one variable,
- $M$ is a primitive monomial of degree 0, *i.e.*, if $M = \prod_{s,j} v_{s,j}^{a_{s,j}}$, then $\gcd(a_{s,j}) = 1$ and $\sum_{s,j} a_{s,j} = 0$.

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The primitive monomials appearing in the factorization of $s_\chi(\mathbf{v})$ are unique up to inversion.

**Schur elements of $G_2$ :**
**$X_0^2 := u_0,\ X_1^2 := -u_1,\ Y_0^2 := w_0,\ Y_1^2 := -w_1.$**

$$s_1 = \Phi_4(X_0X_1^{-1}) \cdot \Phi_4(Y_0Y_1^{-1}) \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_0X_1^{-1}Y_1^{-1})$$

$$s_2 = 2 \cdot X_1^2X_0^{-2} \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_1X_1^{-1}Y_0^{-1})$$

$\Phi_4(x) = x^2 + 1,\ \Phi_3(x) = x^2 + x + 1,\ \Phi_6(x) = x^2 - x + 1.$

## Schur elements of $G_4$ : $X_i^6 := \zeta_3^{-i} u_i$.

$$s_1 = \Phi_9''(X_0X_1^{-1}) \cdot \Phi_{18}'(X_0X_1^{-1}) \cdot \Phi_4(X_0X_1^{-1}) \cdot \Phi_{12}'(X_0X_1^{-1}) \cdot \Phi_{12}''(X_0X_1^{-1}) \cdot \Phi_{36}'(X_0X_1^{-1}) \cdot \Phi_9'(X_0X_2^{-1}) \cdot \Phi_{18}''(X_0X_2^{-1}) \cdot \Phi_4(X_0X_2^{-1}) \cdot \Phi_{12}'(X_0X_2^{-1}) \cdot \Phi_{12}''(X_0X_2^{-1}) \cdot \Phi_{36}''(X_0X_2^{-1}) \cdot \Phi_4(X_0^2X_1^{-1}X_2^{-1}) \cdot \Phi_{12}'(X_0^2X_1^{-1}X_2^{-1}) \cdot \Phi_{12}''(X_0^2X_1^{-1}X_2^{-1})$$

$$s_2 = -\zeta_3^2X_2^6X_1^{-6}\Phi_9'(X_1X_0^{-1}) \cdot \Phi_{18}''(X_1X_0^{-1}) \cdot \Phi_9''(X_2X_0^{-1}) \cdot \Phi_{18}'(X_2X_0^{-1}) \cdot \Phi_4(X_1X_2^{-1}) \cdot \Phi_{12}'(X_1X_2^{-1}) \cdot \Phi_{12}''(X_1X_2^{-1}) \cdot \Phi_{36}'(X_1X_2^{-1}) \cdot \Phi_4(X_0^{-2}X_1X_2) \cdot \Phi_{12}'(X_0^{-2}X_1X_2) \cdot \Phi_{12}''(X_0^{-2}X_1X_2)$$

$$s_3 = \Phi_4(X_0^2X_1^{-1}X_2^{-1}) \cdot \Phi_{12}'(X_0^2X_1^{-1}X_2^{-1}) \cdot \Phi_{12}''(X_0^2X_1^{-1}X_2^{-1}) \cdot \Phi_4(X_1^2X_2^{-1}X_0^{-1}) \cdot \Phi_{12}'(X_1^2X_2^{-1}X_0^{-1}) \cdot \Phi_{12}''(X_1^2X_2^{-1}X_0^{-1}) \cdot \Phi_4(X_2^2X_0^{-1}X_1^{-1}) \cdot \Phi_{12}'(X_2^2X_0^{-1}X_1^{-1}) \cdot \Phi_{12}''(X_2^2X_0^{-1}X_1^{-1})$$

$\Phi_4(x) = x^2+1$, $\Phi_9'(x) = x^3-\zeta_3$, $\Phi_9''(x) = x^3-\zeta_3^2$, $\Phi_{12}''(x) = x^2+\zeta_3$, $\Phi_{12}'(x) = x^2+\zeta_3^2$, $\Phi_{18}''(x) = x^3+\zeta_3$, $\Phi_{18}'(x) = x^3+\zeta_3^2$, $\Phi_{36}''(x) = x^6+\zeta_3$, $\Phi_{36}'(x) = x^6+\zeta_3^2$.

# Cyclotomic Hecke algebras

## Definition

Let $y$ be an indeterminate. A cyclotomic specialization of $\mathcal{H}$ is a $\mathbb{Z}_K$-algebra morphism $\phi : \mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}] \to \mathbb{Z}_K[y, y^{-1}]$ such that

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The corresponding cyclotomic Hecke algebra $\mathcal{H}_\phi$ is the $\mathbb{Z}_K[y, y^{-1}]$-algebra obtained via the specialization of $\mathcal{H}$ via the morphism $\phi$. It also has a symmetrizing form $t_\phi$ defined as the specialization of the form $t$.

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- *The spetsial algebra $\mathcal{H}^s(W)$ is the cyclotomic Hecke algebra obtained via*

$$v_{s,0} \mapsto y,\ \ v_{s,j} \mapsto 1 \textit{ for } 1 \leq j \leq \mathbf{o}(s) - 1, \textit{ for all } s.$$

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The Schur element $s_{\chi_\phi}(y)$ associated to the irreducible character $\chi_\phi$ of $K(y)\mathcal{H}_\phi$ is a Laurent polynomial in $y$ of the form

$$s_{\chi_\phi}(y) = \psi_{\chi,\phi} y^{a_{\chi,\phi}} \prod_{\Phi \in C_K} \Phi(y)^{n_{\chi,\phi}},$$

where $\psi_{\chi,\phi} \in \mathbb{Z}_K$, $a_{\chi,\phi} \in \mathbb{Z}$, $n_{\chi,\phi} \in \mathbb{N}$ and $C_K$ is a set of $K$-cyclotomic polynomials.

# Rouquier blocks

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We call Rouquier ring of $K$ the $\mathbb{Z}_K$-subalgebra of $K(y)$

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| $W$ c.r.g. (non-Weyl) : | Rouquier blocks $\equiv$ ? |

## Proposition

The characters $\chi_\phi$ and $\psi_\phi$ are in the same Rouquier block of $\mathcal{H}_\phi$ if and only if there exist a finite sequence $\chi_0, \chi_1, \ldots, \chi_n \in \mathrm{Irr}(W)$ and a finite sequence $\mathfrak{p}_1, \ldots, \mathfrak{p}_n$ of prime ideals of $\mathcal{R}$ such that

- $(\chi_0)_\phi = \chi_\phi$ et $(\chi_n)_\phi = \psi_\phi$,
- $\forall j \ (1 \leq j \leq n)$, $(\chi_{j-1})_\phi$ et $(\chi_j)_\phi$ are in the same block $\mathcal{R}_{\mathfrak{p}_j}\mathcal{H}_\phi$.

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- $(\chi_0)_\phi = \chi_\phi$ et $(\chi_n)_\phi = \psi_\phi$,
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If $\Omega := \mathbb{Z}_K[y, y^{-1}]$, then $\mathcal{R}_{\mathfrak{p}\mathcal{R}} \simeq \Omega_{\mathfrak{p}\Omega}$ for all prime ideals $\mathfrak{p}$ of $\mathbb{Z}_K$.

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AIM: Determine the blocks $\Omega_{\mathfrak{p}\Omega} \mathcal{H}_\phi$.

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Let $\phi$ be a cyclotomic specialization. A monomial $M$ in $A$ is singular for $\phi$ if $\phi(M) = 1$.

# $\mathfrak{p}$-blocks and $\mathfrak{p}$-essential monomials

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Moreover, the partition $\mathcal{B}_{\mathfrak{p}}^M(\mathcal{H})$ coincides with the blocks of the algebra $A_{\mathfrak{q}_M}\mathcal{H}$, where $\mathfrak{q}_M := (M-1)A + \mathfrak{p}A$.

# The example of $G_2$

We denote the characters of $G_2$ as follows:

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**Schur elements: 2-essential in purple, 3-essential in green**

$$s_1 = \Phi_4(X_0X_1^{-1}) \cdot \Phi_4(Y_0Y_1^{-1}) \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_0X_1^{-1}Y_1^{-1})$$

$$s_2 = 2 \cdot X_1^2X_0^{-2} \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_1X_1^{-1}Y_0^{-1})$$

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| **Monomial** | $\mathcal{B}_2^M(\mathcal{H})$ | $\mathcal{B}_3^M(\mathcal{H})$ |
|---|---|---|
| 1 | $(\chi_{2,1}, \chi_{2,2})$ | - |
| $M_1$ | $(\chi_{1,0}, \chi_{1,3'}), (\chi_{2,1}, \chi_{2,2}), (\chi_{1,6}, \chi_{1,3''})$ | - |
| $M_2$ | $(\chi_{1,0}, \chi_{1,3''}), (\chi_{2,1}, \chi_{2,2}), (\chi_{1,6}, \chi_{1,3'})$ | - |
| $M_3$ | $(\chi_{2,1}, \chi_{2,2})$ | $(\chi_{1,0}, \chi_{1,6}, \chi_{2,2})$ |
| $M_4$ | $(\chi_{2,1}, \chi_{2,2})$ | $(\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1})$ |

# Determination of the Rouquier blocks of a cyclotomic Hecke algebra

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