Generalizing Hook Lengths for Partitions

Maria Chlouveraki

National & Kapodistrian University of Athens

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Representation Theory

Algebraic Combinatorics

$\mathrm{Irr}(\mathbb{C}S_n)$

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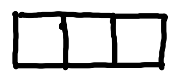

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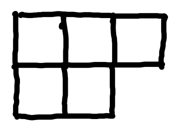

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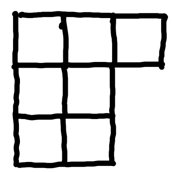

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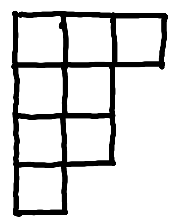

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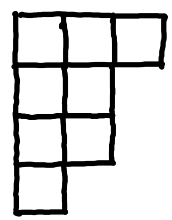

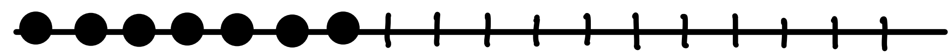

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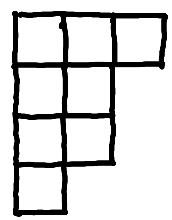

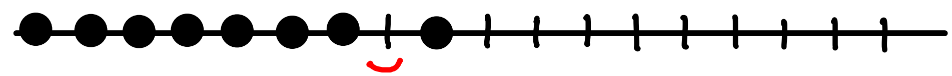

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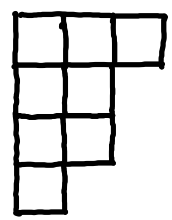

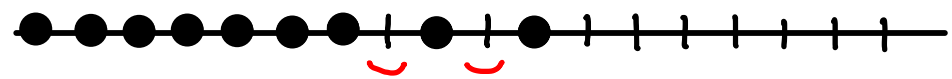

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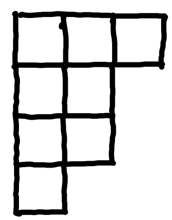

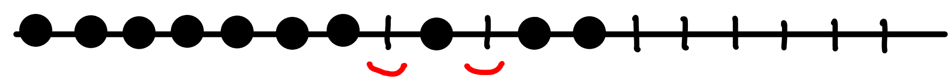

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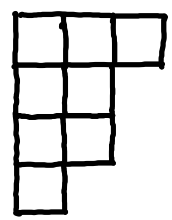

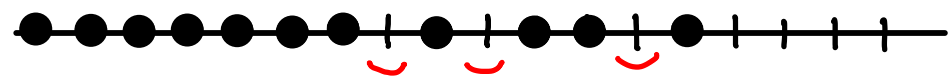

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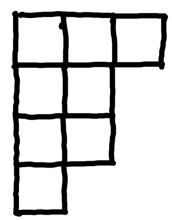

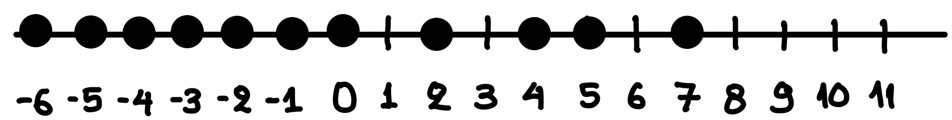

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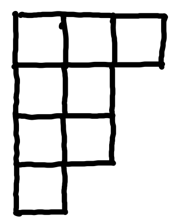

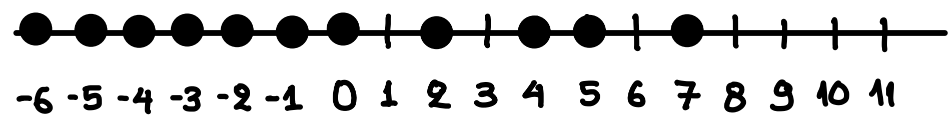

Hook length of $(i,j) \in Y(\lambda)$

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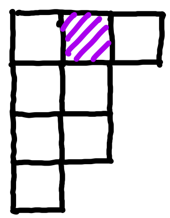

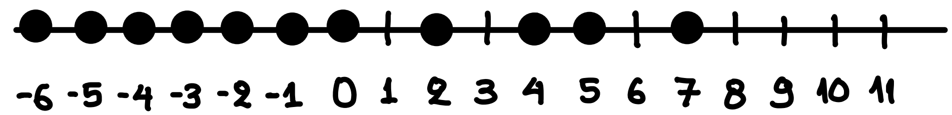

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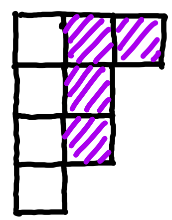

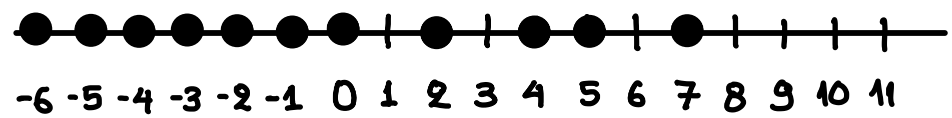

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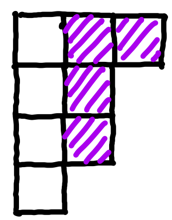

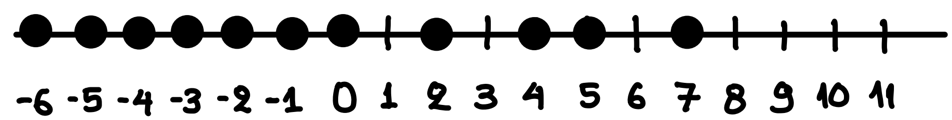

Hook length of $(i,j) \in Y(\lambda)$

<u>Ex.</u> $h_{1,2} = 4$

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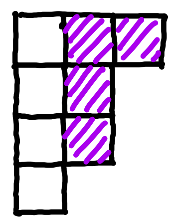

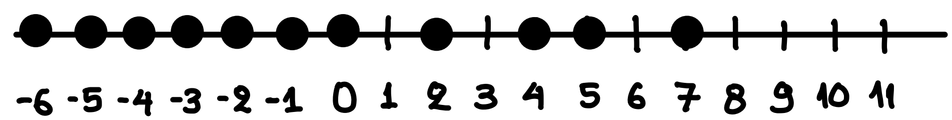

Hook length of $(i,j) \in Y(\lambda)$

$$\boxed{h_{i,j} = \lambda_i - i + \lambda'_j - j + 1}$$

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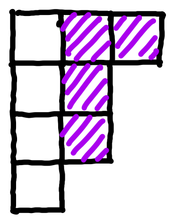

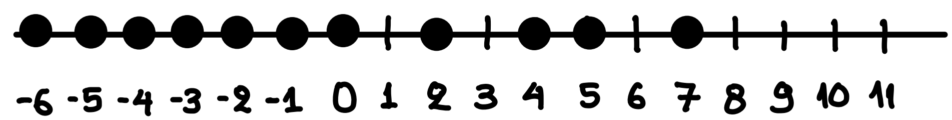

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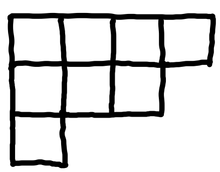

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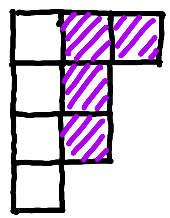

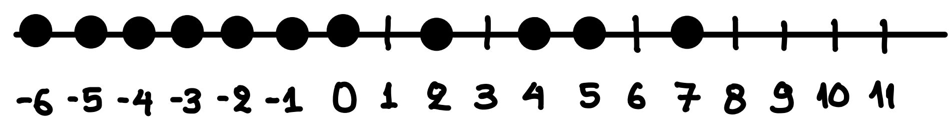

Hook length of $(i,j) \in Y(\lambda)$ $\boxed{h_{i,j} = \lambda_i - i + \lambda'_j - j + 1}$

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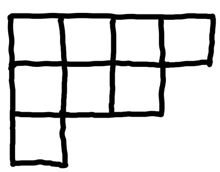

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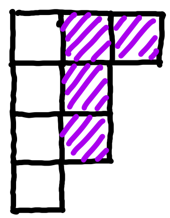

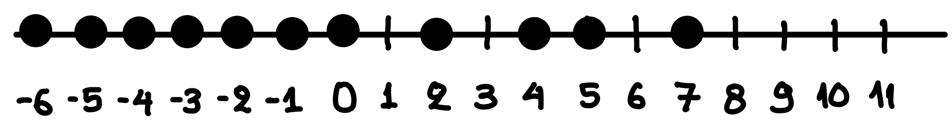

Hook length of $(i,j) \in Y(\lambda)$ $\quad \boxed{h_{i,j} = \lambda_i - i + \lambda'_j - j + 1}$

<u>Ex.</u> $h_{1,2} = 3-1+3-2+1 = 7-3 = 4$

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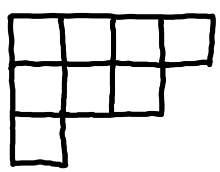

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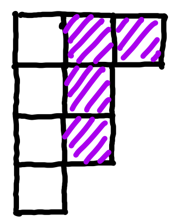

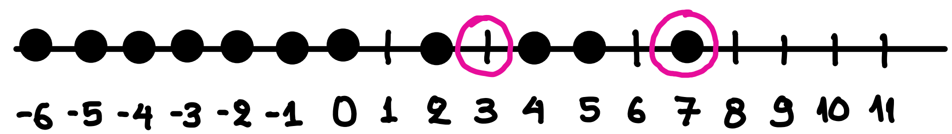

Hook length of $(i,j) \in Y(\lambda)$

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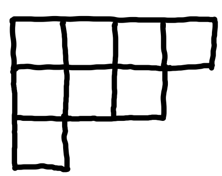

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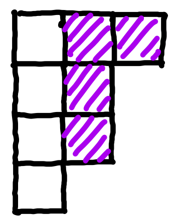

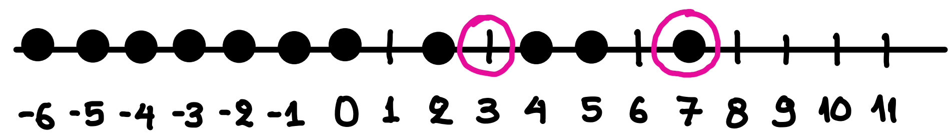

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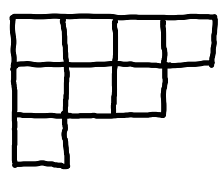

Hook length formula

$$\boxed{\dim V_\lambda = \frac{n!}{\prod_{i,j} h_{i,j}}}$$

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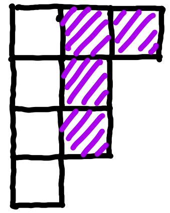
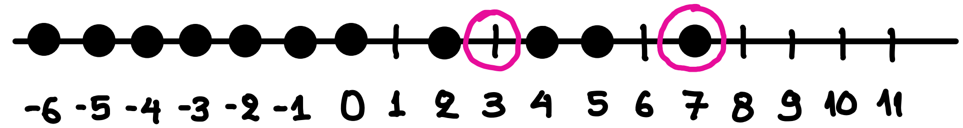


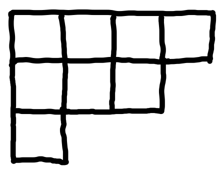


Hook length of $(i,j) \in Y(\lambda)$

$$\boxed{h_{i,j} = \lambda_i - i + \lambda'_j - j + 1}$$

Ex. $h_{1,2} = 3-1+3-2+1 = 7-3 = 4$

$\lambda' = (4, 3, 1)$



Hook length formula

$$\boxed{\dim V_\lambda = \frac{n!}{\prod_{i,j} h_{i,j}}}$$

Ex. Hook lengths are: $7-1$, $7-3$, $7-6$, $5-1$, $5-3$, $4-1$, $4-3$, $2-1$

$$\dim V_\lambda = \frac{8!}{6\cdot 4\cdot 1\cdot 4\cdot 2\cdot 3\cdot 1\cdot 1} = 70$$

$$\mathrm{Irr}(kS_n) \longleftrightarrow \{\text{Partitions of } n\}$$

when $\operatorname{char} k \nmid n!$

$$\mathrm{Irr}(kS_n) \longleftrightarrow \{\text{p-regular partitions of } n\}$$

when $\operatorname{char} k = p \mid n!$

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BLOCKS: V_λ and V_μ are in the same block B

$\Updownarrow$

λ and μ have the same p-core

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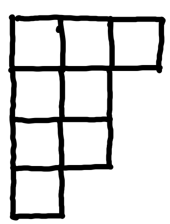

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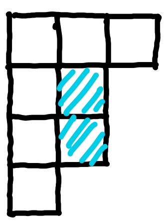

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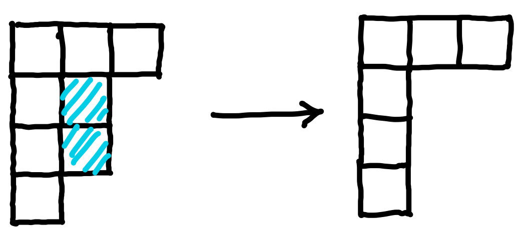

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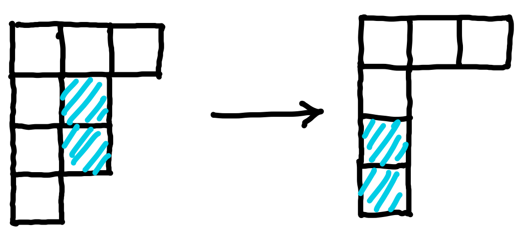

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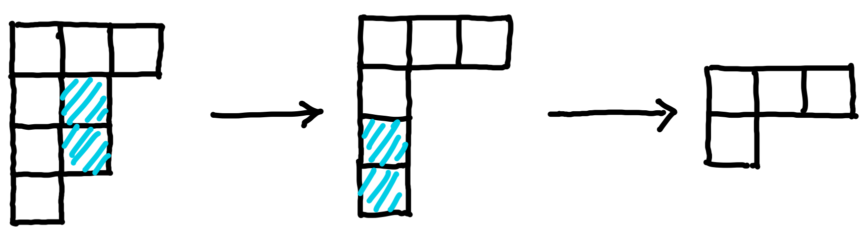

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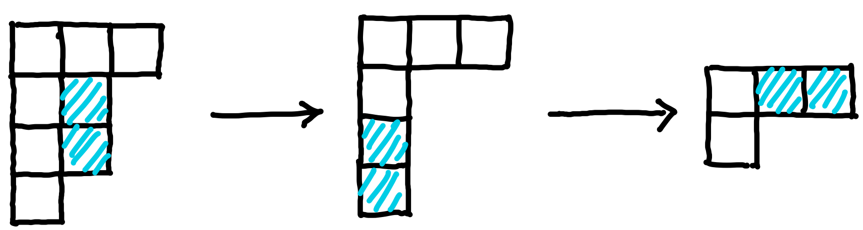

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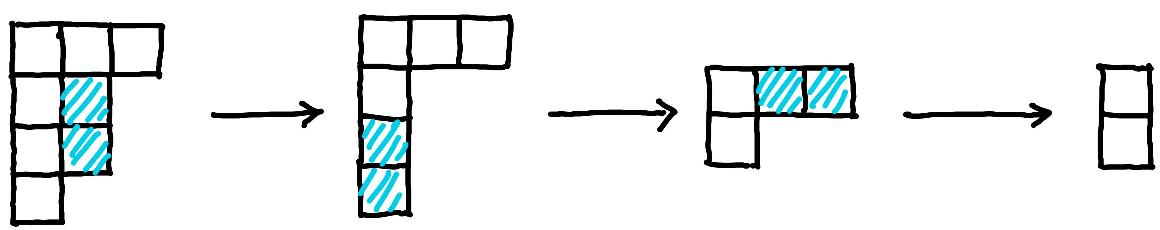

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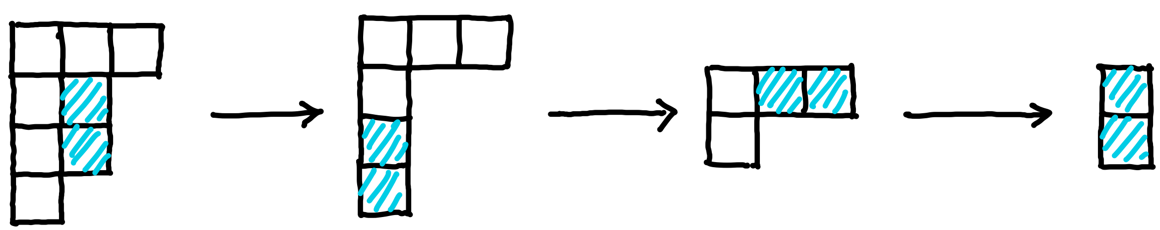

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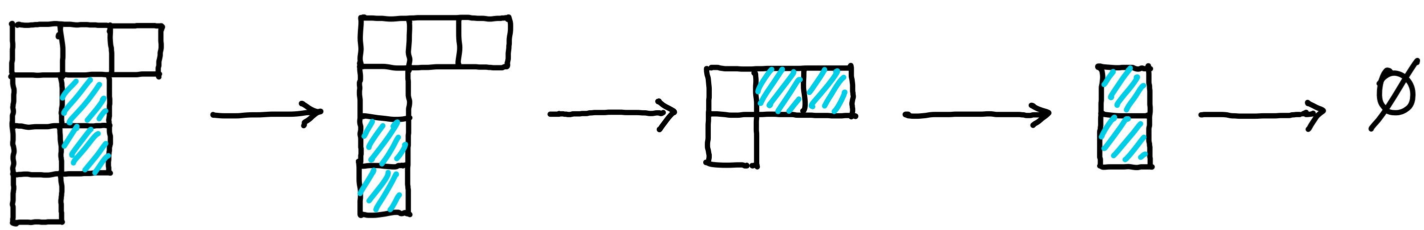

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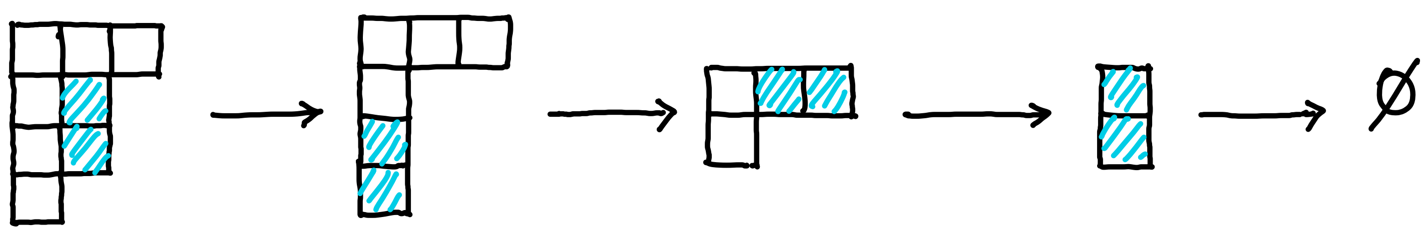

Defect of λ: $d_\lambda = v_p\left(\prod_{i,j} h_{i,j}\right)$

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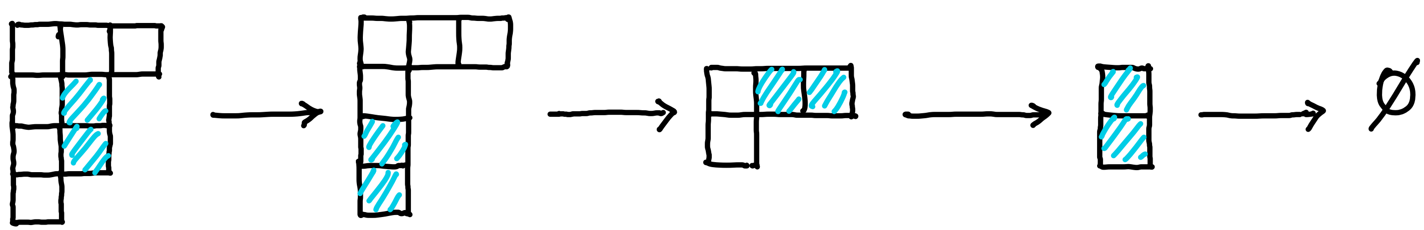

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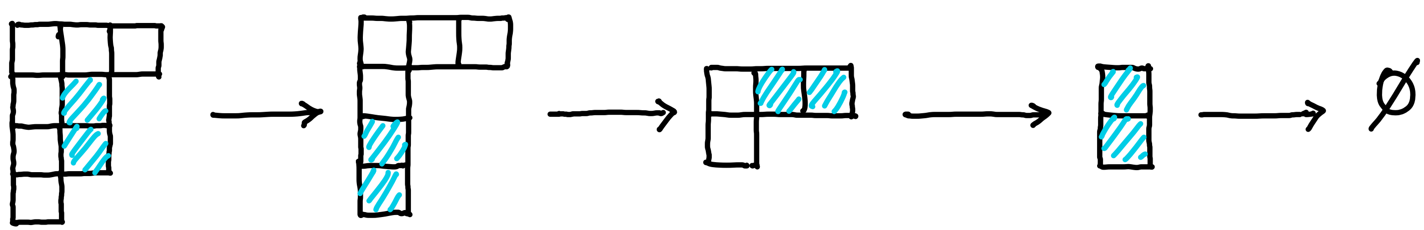

Defect of λ : $d_\lambda = v_p\left(\prod_{i,j} h_{i,j}\right)$ $\qquad$ $d(B) = \max\{d_\lambda \mid V_\lambda \in B\}$

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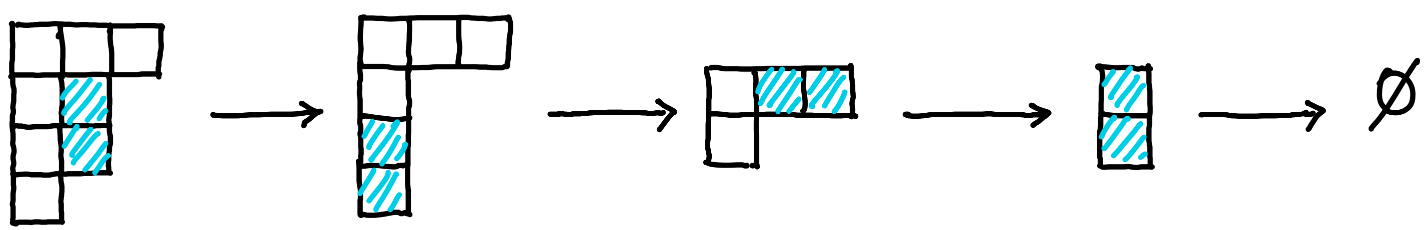

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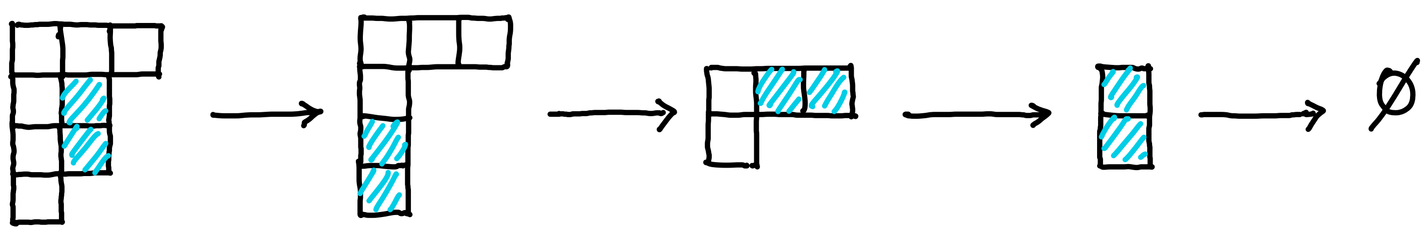

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Block Invariant

Ex. $\lambda = (3,2,2,1)$ $HL(\lambda) = \{\{ 6,4,4,3,2,2,1,1,1 \}\}$

$d_\lambda = v_2(2^7 \cdot 3^2) = 7$

$\mathrm{Irr}(kS_n) \longleftrightarrow \{\text{ p-regular partitions of } n\}$

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Def^n : $\prod_{i,j} h_{i,j} = \dfrac{|S_n|}{\dim V_\lambda}$ is the Schur element s_λ of λ

$$\mathbb{C}S_n = \left\langle s_1, \dots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j| > 1 \\ s_i^2 = 1 \end{array} \right\rangle$$

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1, \ldots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j| > 1 \\ s_i^2 = (q-1) s_i + q \end{array} \right\rangle$$

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$$\mathrm{Irr}(\mathbb{C}(q)\mathcal{H}_n(q)) \longleftrightarrow \mathrm{Irr}(\mathbb{C}S_n)$$

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$$\mathrm{Irr}(\mathbb{C}(q)\mathcal{H}_n(q)) \longleftrightarrow \mathrm{Irr}(\mathbb{C}S_n)$$

when q is not a root of unity of order $\leq n$

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1, \dots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j| > 1 \\ s_i^2 = (q-1) s_i + q \end{array} \right\rangle$$

$$s_\lambda = \frac{q^{-a_\lambda}}{(q-1)^n} \prod_{i,j} \left(q^{h_{i,j}} - 1\right)$$

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$$d_\lambda = v_{\phi_e}(s_\lambda) = \#\{(i,j) : e \mid h_{i,j}\}$$

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<u>Thm</u> (Geck'92) : The defect is a block invariant

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1, \ldots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j|>1 \\ s_i^2 = (q-1)s_i + q \end{array} \right\rangle$$

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Thm (Geck'92) : The defect is a block invariant

Conj (Geck'92) : This holds for all real reflection groups

Defⁿ $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \dots, \lambda^{(d)})$ d-partition of n if $\forall\, i = 1, \dots, d$

$\lambda^{(i)}$ is a partition and $\sum |\lambda^{(i)}| = n$

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<u>Ex.</u> $\left(\ydiagram{2,2} \, , \, \ydiagram{1,1} \right)$ is a 2-partition of 6

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$\{ d\text{-partitions of } n \} \longleftrightarrow \mathrm{Irr}(\mathbb{C}G(d,1,n))$

where $G(d,1,n) = (\mathbb{Z}/d\mathbb{Z}) \rtimes S_n$

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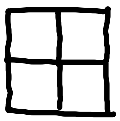
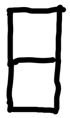
Ex. ( , ) is a 2-partition of 6

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For $d=1$, $G(1,1,n) = S_n$

$d=2$, $G(2,1,n) = B_n$

$\underline{\text{Def}^{n}}$ $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \dots, \lambda^{(d)})$ d-partition of n if $\forall\, i = 1, \dots, d$ $\lambda^{(i)}$ is a partition and $\sum |\lambda^{(i)}| = n$

$\underline{\text{Ex.}}$ $\left(\ydiagram{2,2} \,,\, \ydiagram{1,1} \right)$ is a 2-partition of 6

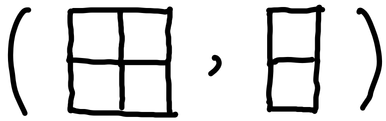
$$\{ d\text{-partitions of } n \} \longleftrightarrow \operatorname{Irr}(\mathbb{C}G(d,1,n)) \longleftrightarrow \operatorname{Irr}(\mathbb{C}(q)\mathcal{H}_{d,n}(q))$$

where $G(d,1,n) = (\mathbb{Z}/d\mathbb{Z}) \rtimes S_n$

For $d = 1$, $G(1,1,n) = S_n$

$d = 2$, $G(2,1,n) = B_n$

Def^n $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \dots, \lambda^{(d)})$ d-partition of n if $\forall\, i = 1, \dots, d$ $\lambda^{(i)}$ is a partition and $\sum |\lambda^{(i)}| = n$

Ex.  is a 2-partition of 6

$$\{\text{d-partitions of } n\} \longleftrightarrow \mathrm{Irr}(\mathbb{C}G(d,1,n)) \longleftrightarrow \mathrm{Irr}(\mathbb{C}(q)\mathcal{H}_{d,n}(q))$$

where $G(d,1,n) = (\mathbb{Z}/d\mathbb{Z}) \rtimes S_n$

Def^n (C.- 07/01/11)

$(i,j) \in Y(\lambda^{(a)})$

$$\boxed{h^{a,b}_{i,j} = \lambda^{(a)}_i - i + \lambda^{(b)\prime}_j - j + 1}$$

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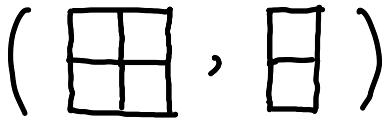
Ex.  is a 2-partition of 6

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Def^n (C. - 07/01/11)
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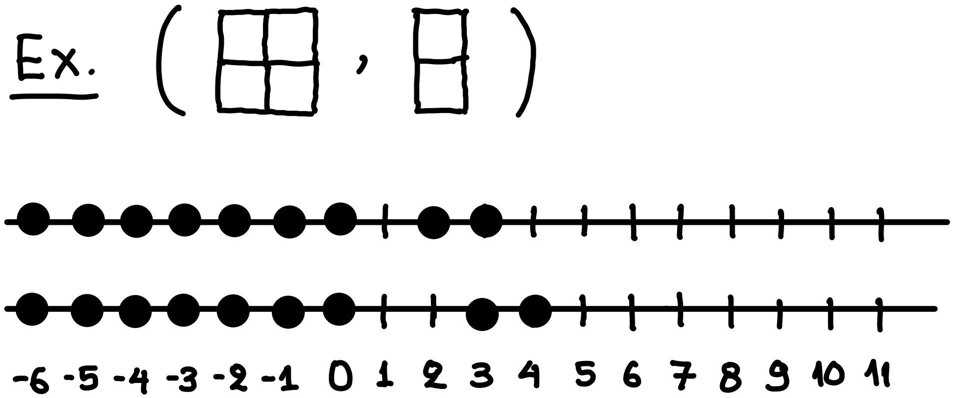
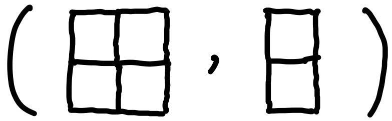
$$\boxed{h_{i,j}^{a,b} = \lambda_i^{(a)} - i + \lambda_j^{(b)'} - j + 1}$$

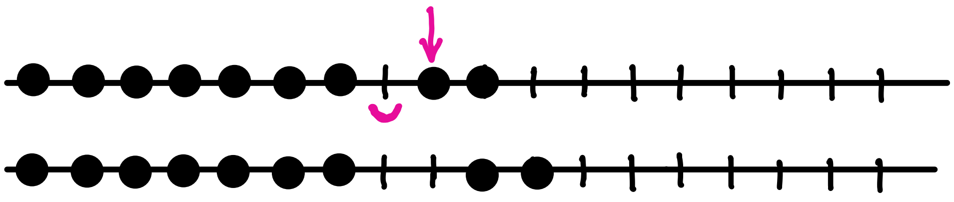
Thm (C.)

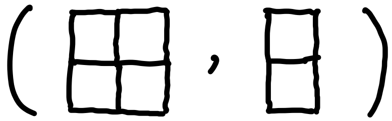
$$S_\lambda = \frac{q^{-a_\lambda}}{(q-1)^n} \prod_{a,b} \prod_{i,j} \left(q^{h_{i,j}^{a,b}} - 1\right)$$

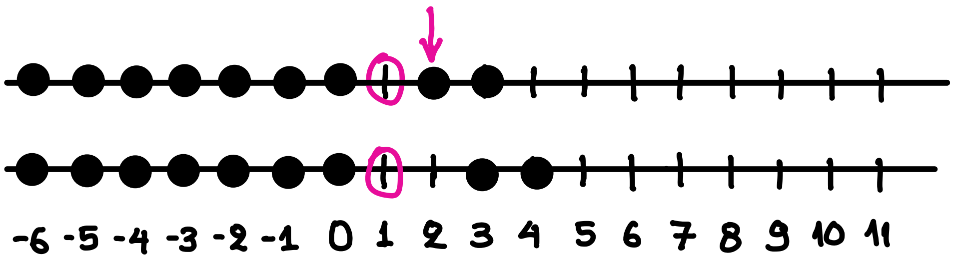
Ex.

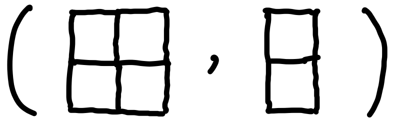


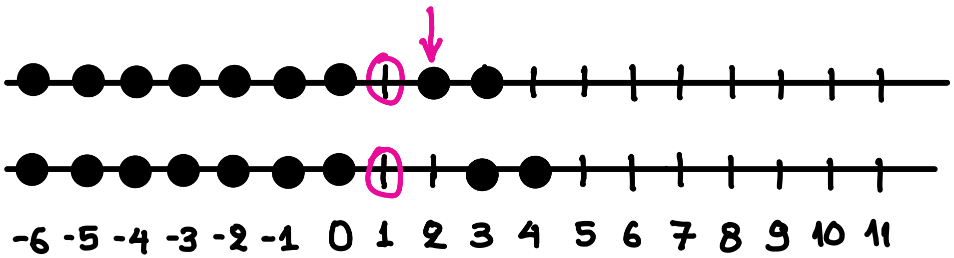
Ex. 



Ex. 

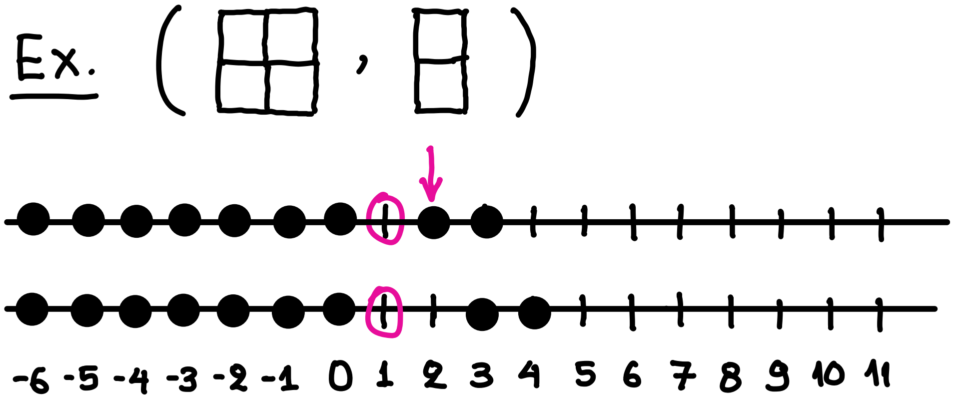


Ex. 



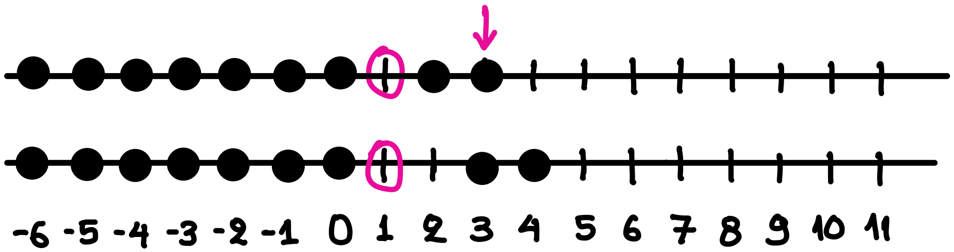
$HL^{CJ}(\lambda) = \{\{\, 2-1 ,\ 2-1$

Ex.



$HL^{CJ}(\lambda) = \{\{ 1, 1$

Ex. $\left(\yng(2,2) \, , \, \yng(1,1) \right)$



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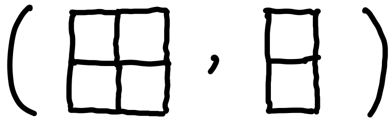
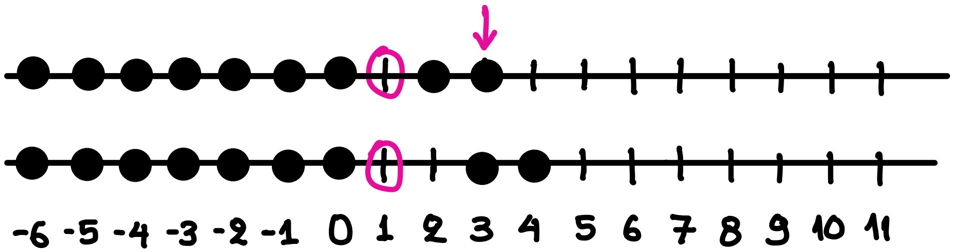
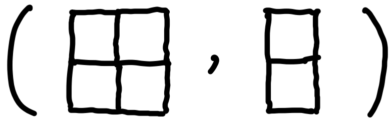
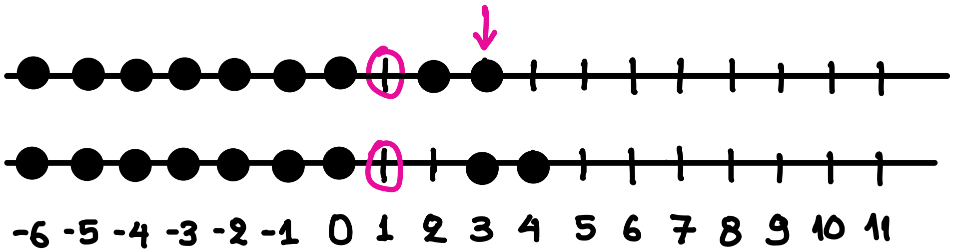
Ex. 

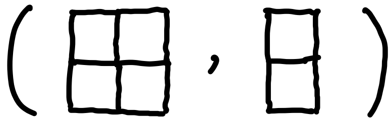


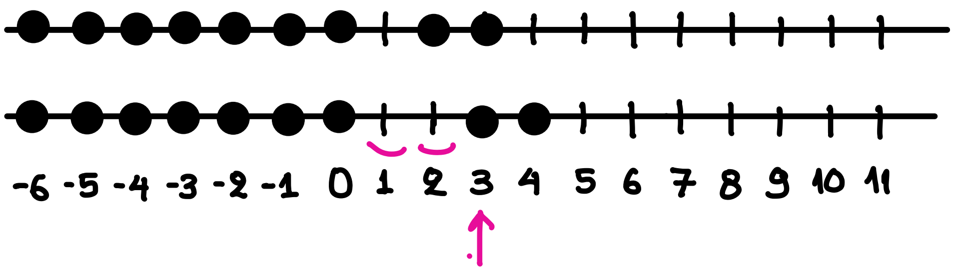
$HL^{CJ}(\lambda) = \{\{\ 1,\ 1,\ 3-1,\ 3-1$

Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2$

Ex. 



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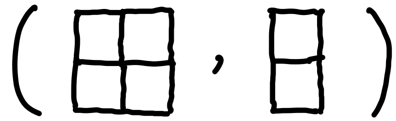
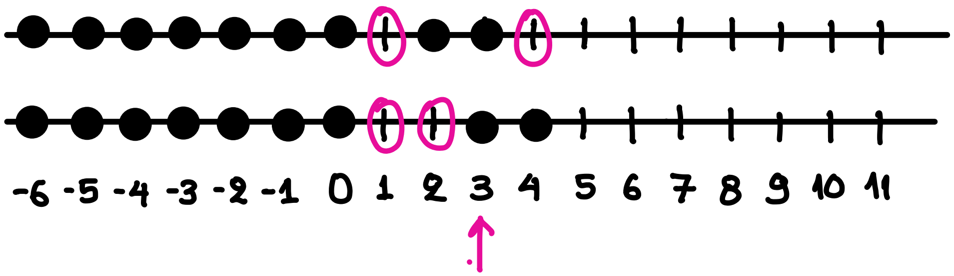
Ex. 



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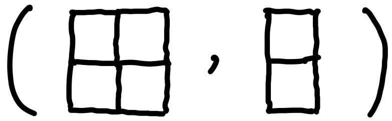
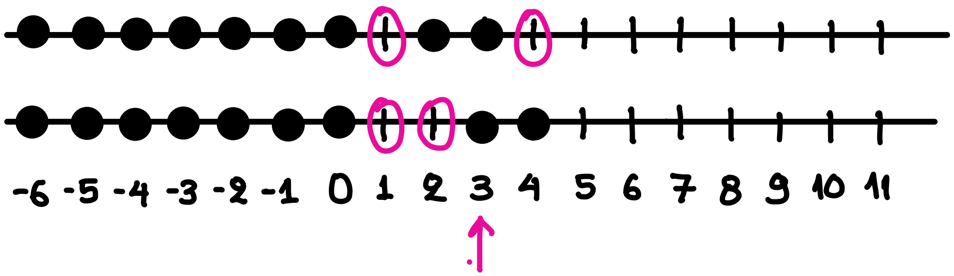
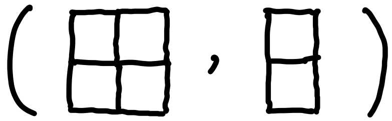
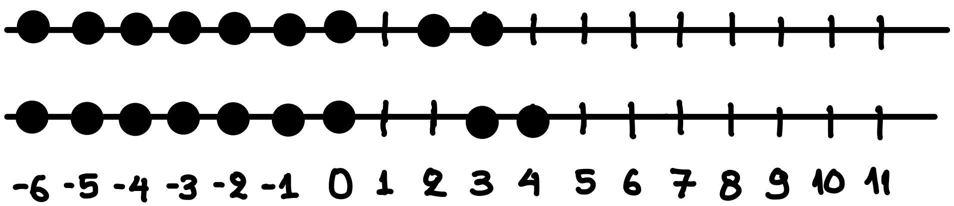
Ex. 

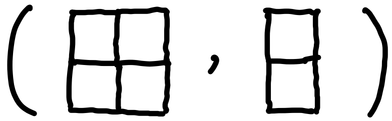


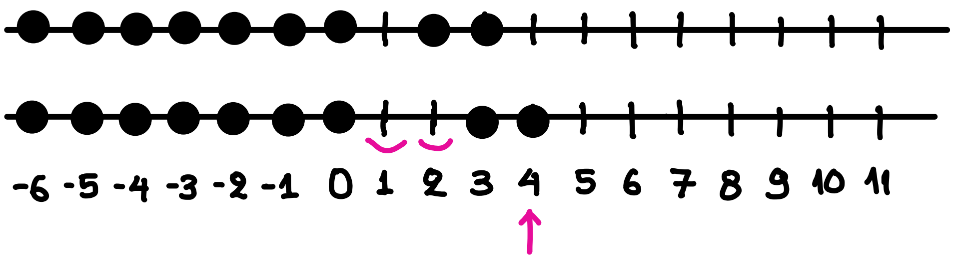
$HL^{CJ}(\lambda) = \{\{\, 1, 1, 2, 2, 3-1, 3-2, 3-1, 3-4$

Ex. 



$HL^{CJ}(\lambda) = \{\{ \; 1, 1, 2, 2, \; 2, 1, 2, -1$

Ex. 



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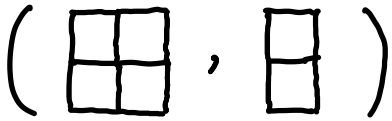
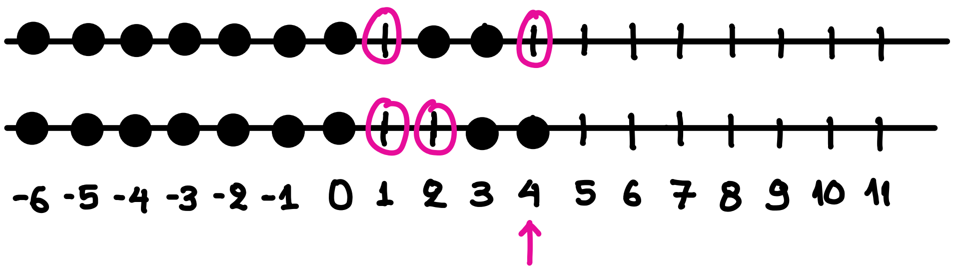
Ex. 



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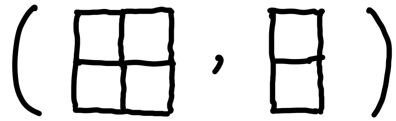
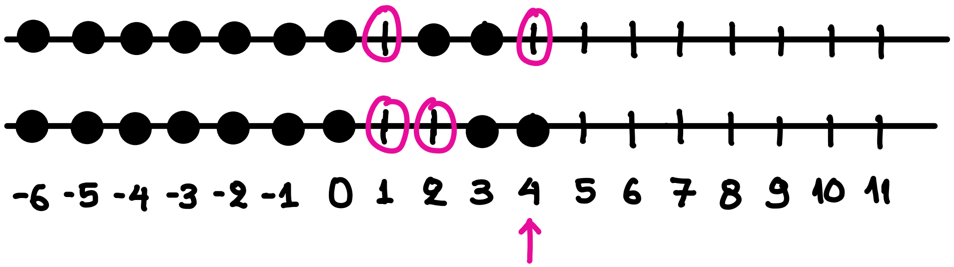
Ex. 



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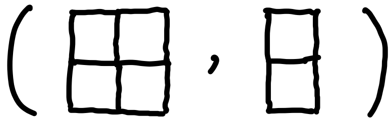
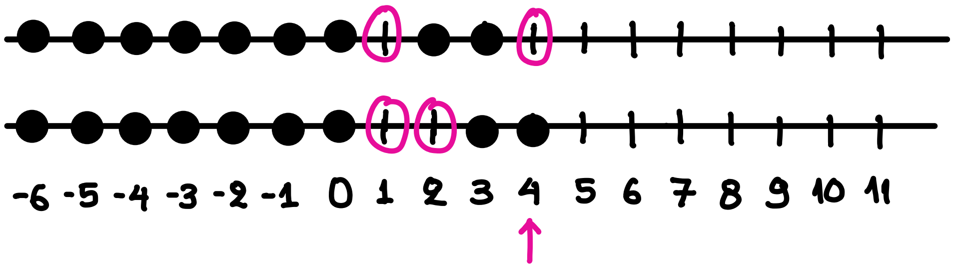
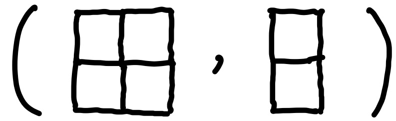
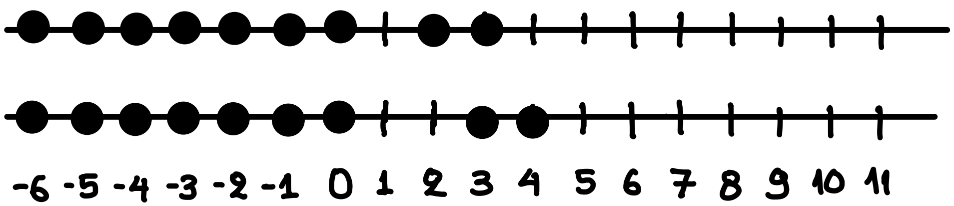
Ex. 



$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

Ex. 



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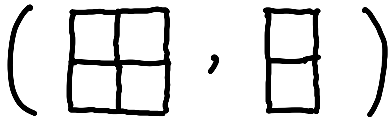
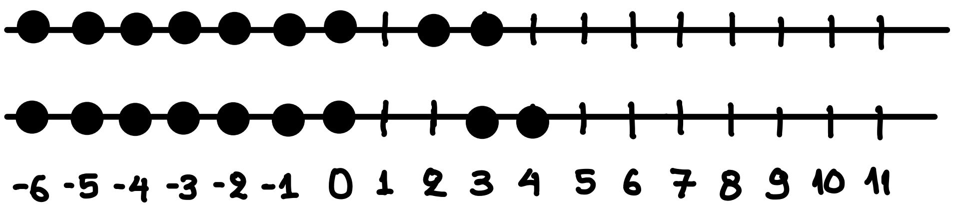
Let q be a root of unity of order $2 \leq e \leq n$

$d_\lambda = \upsilon_{\Phi_e}(s_\lambda) = \#\{(i,j) : e \mid h_{i,j}^{a,b}\}$

Ex. 



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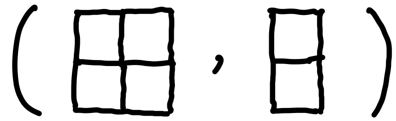
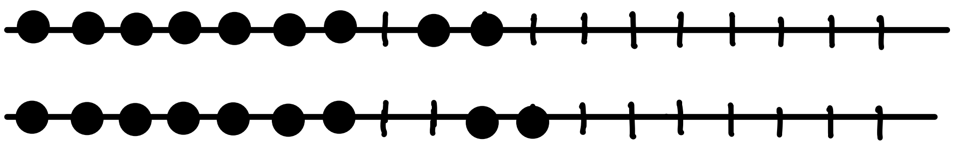
Thm (C.-Jacon'23) : The defect is a block invariant

Ex. 



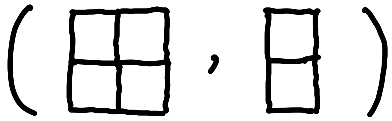
$$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, \; 2, 1, 2, -1, 3, 2, 3, 0 \}\}$$

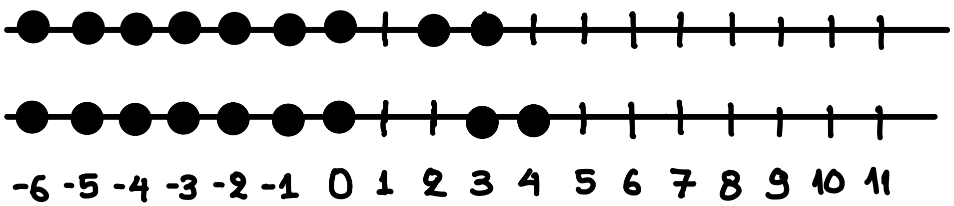
Let q be a root of unity of order $2 \leq e \leq n$

$$d_\lambda = v_{\Phi_e}(s_\lambda) = \# \{ (i,j) : e \mid h_{i,j}^{a,b} \}$$

Thm (C.-Jacon'23) : The defect is a block invariant

Conj (C.-Jacon'23) : This holds for all complex reflection groups

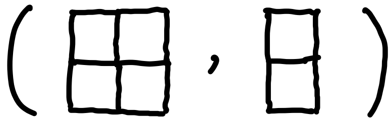
Ex. 

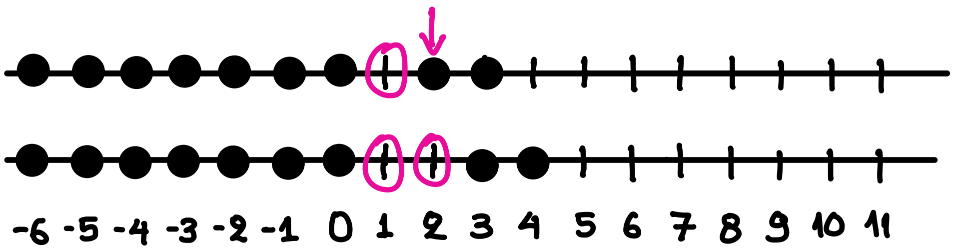


$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

Def^n (Bessenrodt - Gramain - Olsson, 26/01/11)

$HL^{BGO}(\lambda) = \{\{$

Ex. 



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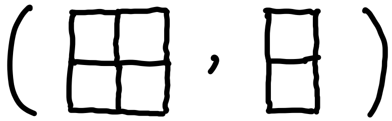
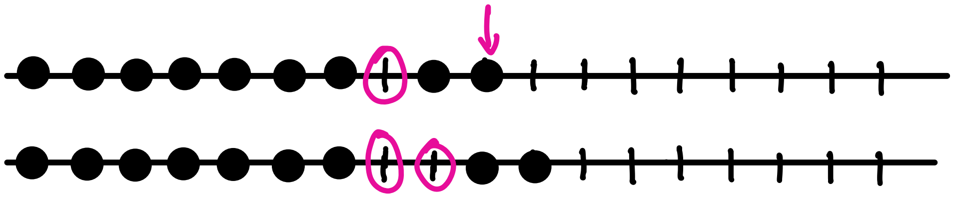
Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

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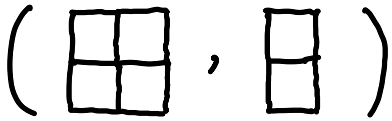
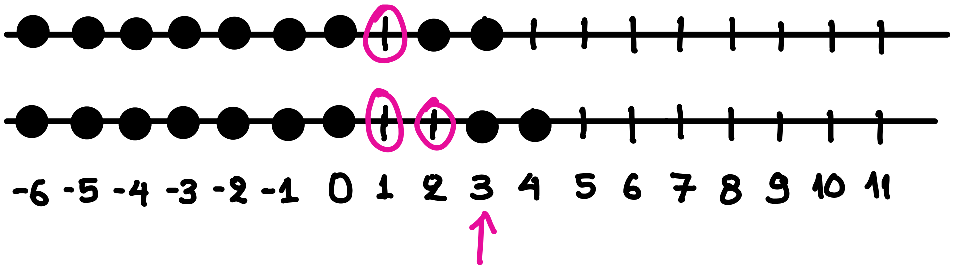
Ex. 



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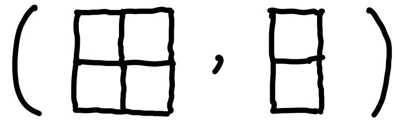
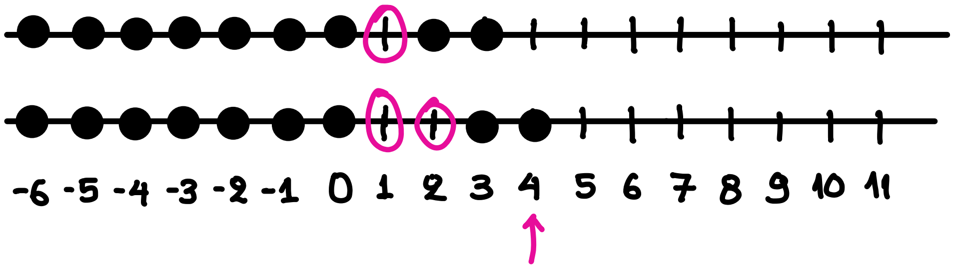
Ex. 



$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

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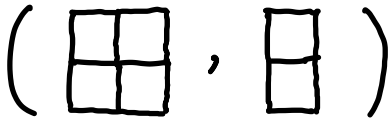
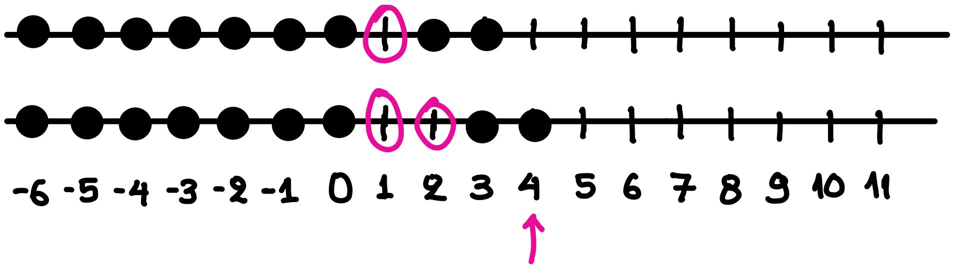
Ex. 



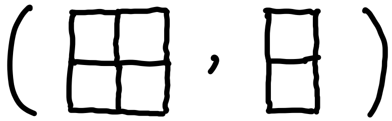
$$HL^{CJ}(\boldsymbol{\lambda}) = \{\{1,1,2,2,2,1,2,-1,3,2,3,0\}\}$$

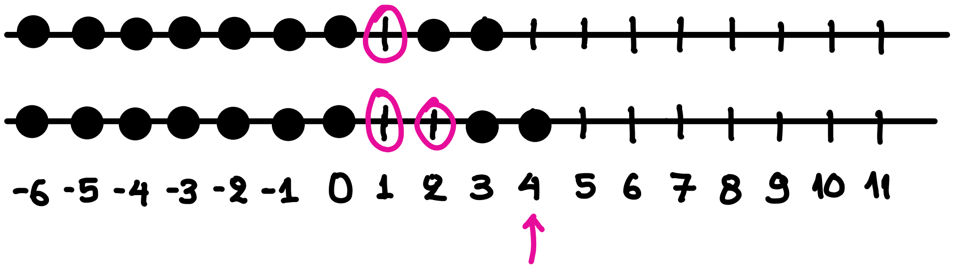
Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

$$HL^{BGO}(\boldsymbol{\lambda}) = \{\{1,1,0,2,2,1,2,2,1,3,3,2\}\}$$

Thm (C. – Gramain – Jacon, '25)

There exists a bijection $HL^{CJ}(\boldsymbol{\lambda}) \longrightarrow HL^{BGO}(\boldsymbol{\lambda})$, $h \longmapsto |h|$

Ex. 



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Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

$$HL^{BGO}(\lambda) = \{\{ 1, 1, 0, 2, 2, 1, 2, 2, 1, 3, 3, 2 \}\}$$

Thm (C. – Gramain – Jacon, '25)

There exists a bijection $HL^{CJ}(\lambda) \longrightarrow HL^{BGO}(\lambda)$, $h \longmapsto |h|$

(when the charges are equal)

<u>Thm</u> (C.-Gramain-Jacon, '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

$$\boxed{s_\lambda = s_{e\text{-core of }\lambda} \cdot s_{e\text{-quotient of }\lambda}}$$

Thm (C.-Gramain-Jacon, '25)

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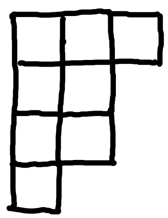
Ex $\lambda =$ 

<u>Thm</u> (C.- Gramain - Jacon , '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

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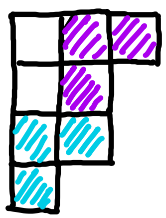
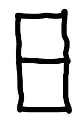
<u>Ex</u> $\lambda =$  3- core of $\lambda =$ 

Thm (C.–Gramain–Jacon, '25)

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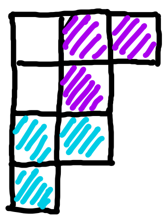
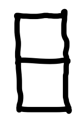
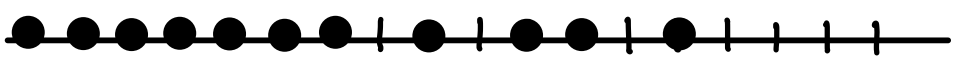
Ex $\lambda =$  3-core of $\lambda =$ 



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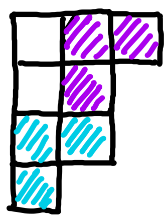
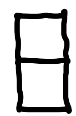
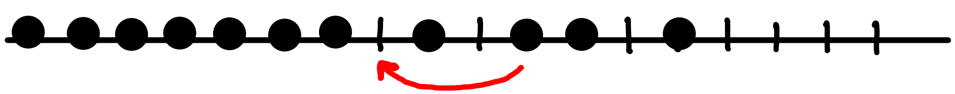
Ex $\lambda =$  3-core of $\lambda =$ 



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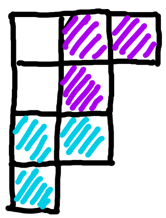
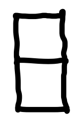
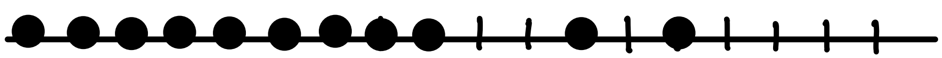
<u>Ex</u> $\lambda =$  3-core of $\lambda =$ 



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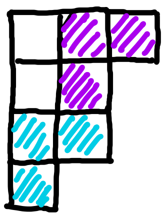
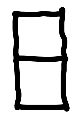
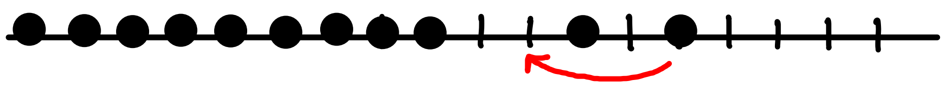
<u>Ex</u> $\lambda =$  3-core of $\lambda =$ 



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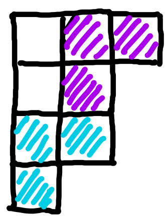
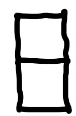
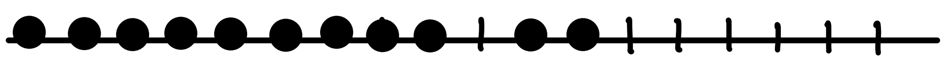
Ex $\lambda =$  3-core of $\lambda =$ 



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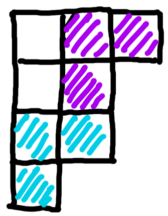
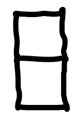
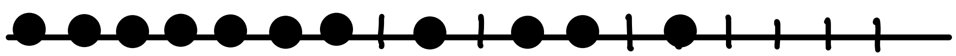
<u>Ex</u> $\lambda =$  3-core of $\lambda =$ 



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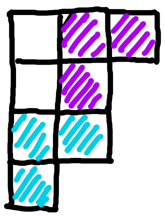
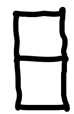
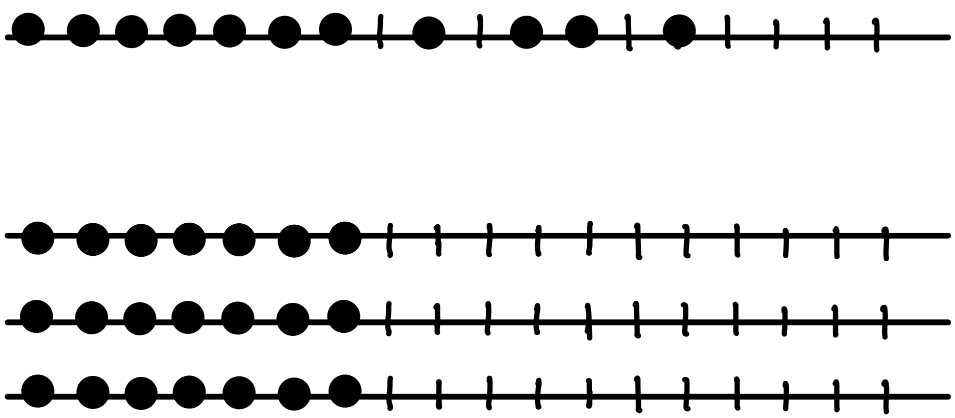
Ex $\lambda =$  3-core of $\lambda =$ 



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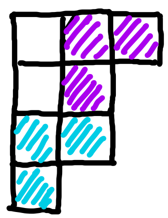
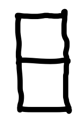
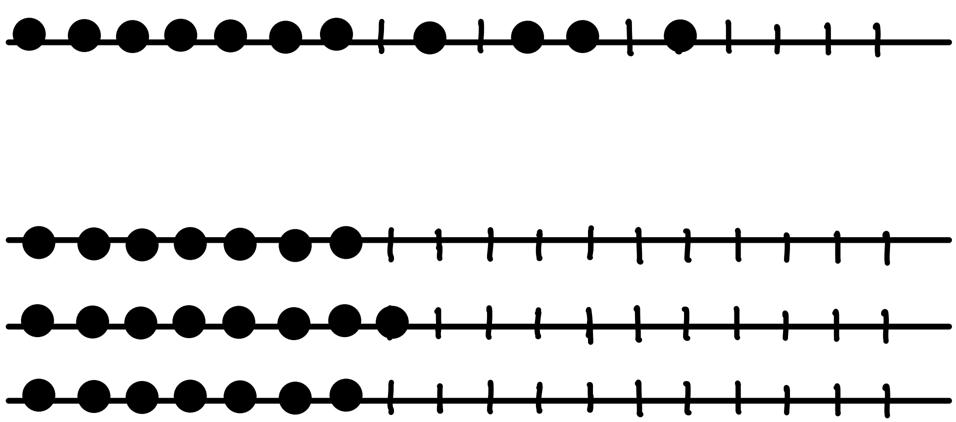
Ex $\lambda =$  3-core of $\lambda =$ 



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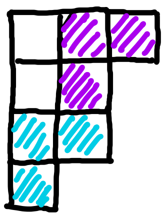
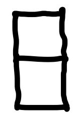
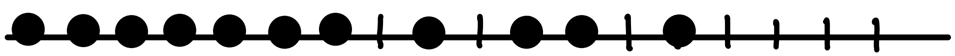
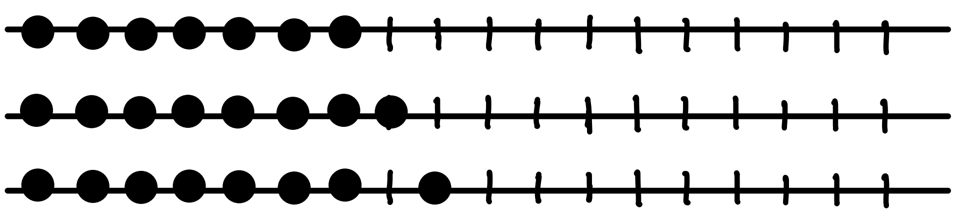
Ex $\lambda =$  3-core of $\lambda =$ 





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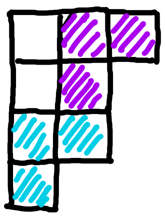
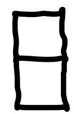
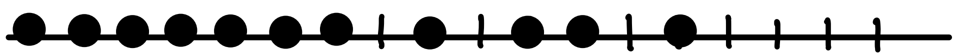
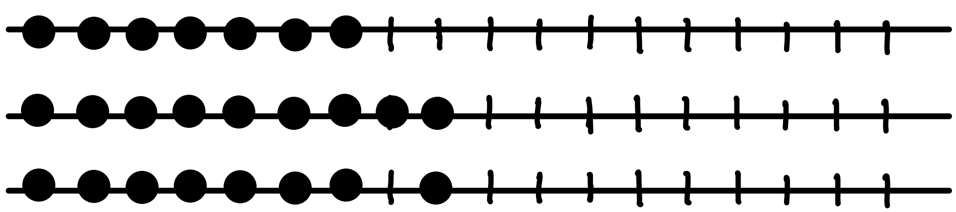
Ex $\lambda =$  3-core of $\lambda =$ 





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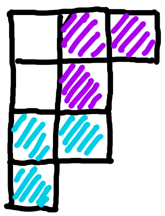
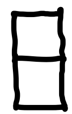
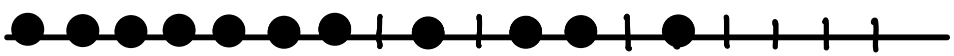
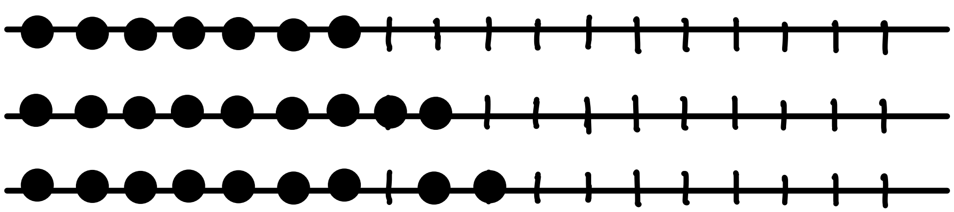
Ex $\lambda =$  3-core of $\lambda =$ 





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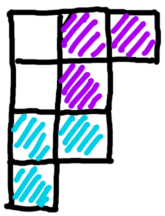
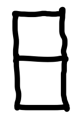
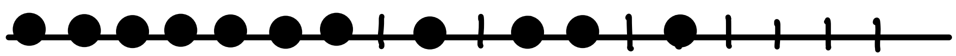
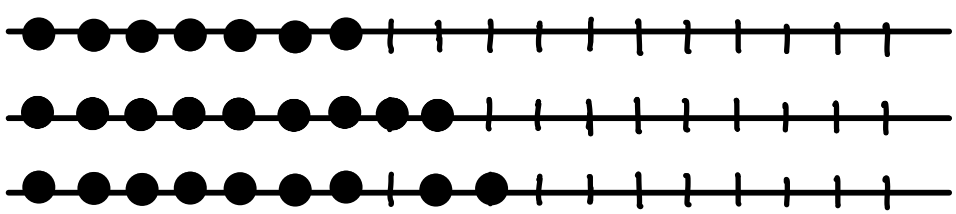
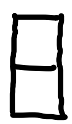
Ex $\lambda =$  3-core of $\lambda =$ 





3-quotient of $\lambda = ($  $, \emptyset, \emptyset)$

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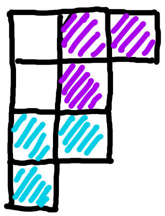
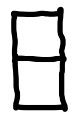
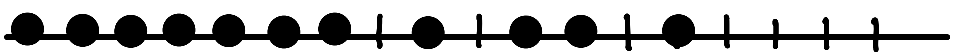
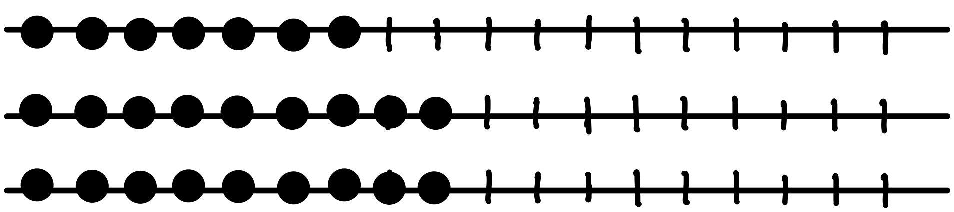
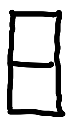
Ex $\lambda =$  3-core of $\lambda =$ 





3-quotient of $\lambda = \Big($  $, \emptyset, \emptyset \Big)$

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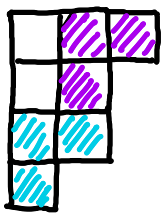
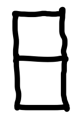
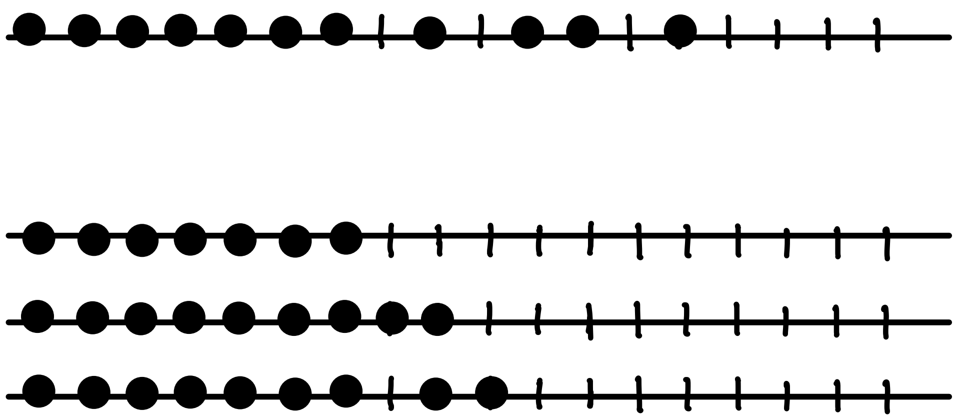
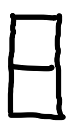
Ex $\lambda =$  3-core of $\lambda =$ 



3-quotient of $\lambda = ($  $, \emptyset, \emptyset)$