Schur elements & Blocks of Hecke algebras

Maria Chlouveraki

University of Athens

Conference on "Representations of Reductive Groups"

Celebrating the 60th birthday of Meinolf Geck

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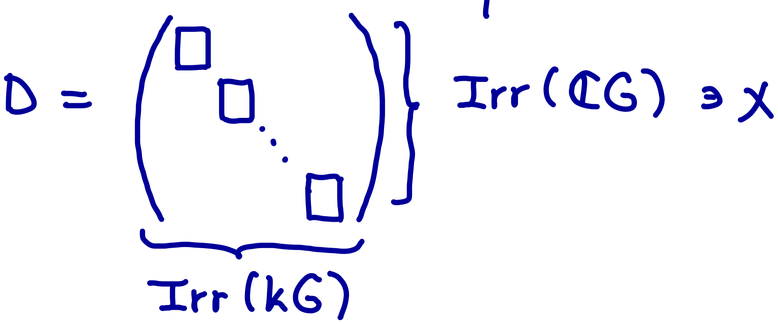

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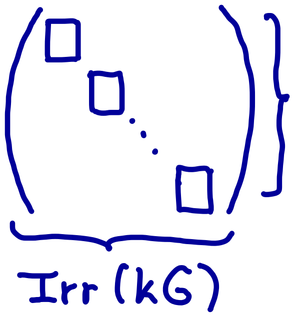

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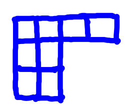
<u>Ex</u> $\lambda =$ 

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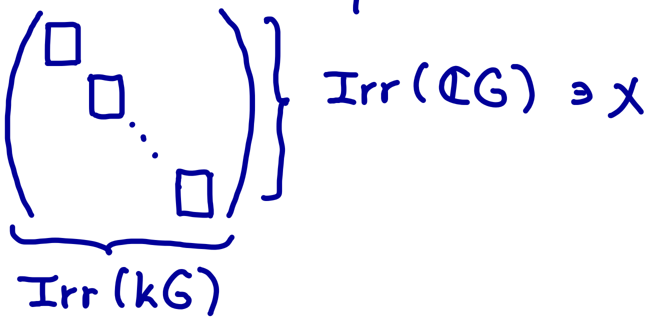

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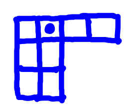
<u>Ex</u> $\lambda =$ 

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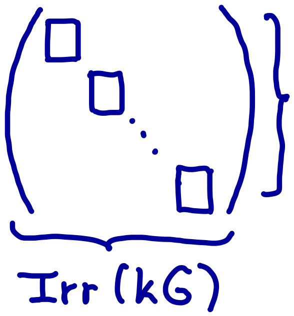

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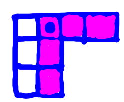

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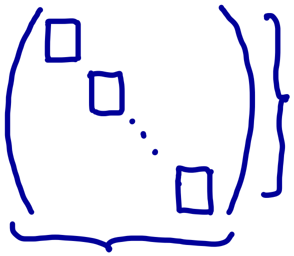
$D =$  $\}\ \operatorname{Irr}(\mathbb{C}G) \ni \chi$

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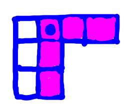
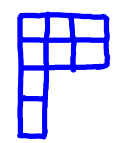
<u>Ex</u> $\lambda =$  $\rightsquigarrow \lambda' =$ 

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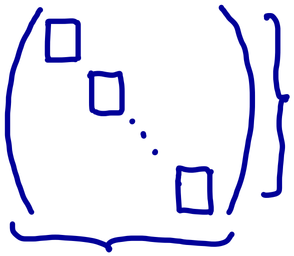
$D =$  $\} \operatorname{Irr}(\mathbb{C}G) \ni \chi$

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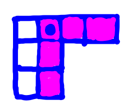
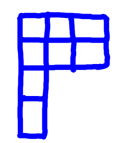
Ex $\lambda =$  $\rightsquigarrow$ $\lambda' =$ 

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BLOCK INVARIANT

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The defect is a block invariant for Hecke algebras of real reflection groups in the equal parameter case.

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Dipper - Mathas, Rostam

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$$h_{i,j} = \lambda_i - i + \lambda_j' - j + 1$$

(Cyclotomic) Hecke algebras of complex reflection groups

$W = \langle S \mid \text{braid relations}, \ s^{n_s} = 1 \rangle$

$\mathcal{H}_q(W) = \langle (T_s)_{s \in S} \mid \text{braid relations} \ \prod_{j=0}^{n_s - 1} (T_s - \zeta_{n_s}^j q^{m_{s,j}}) = 0 \rangle$

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The BMM trace conjecture

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$\lambda^2 = (2,1,1,1)$

$m_2 = 1$

$\lambda^1 = (2,2)$

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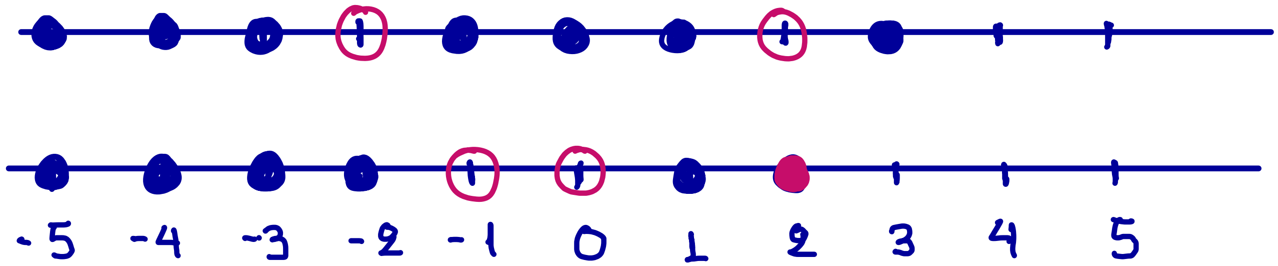

$H(\lambda) = \{ \; 2-(-1), \; 2-0, \; 2-(-2), \; 2-2$

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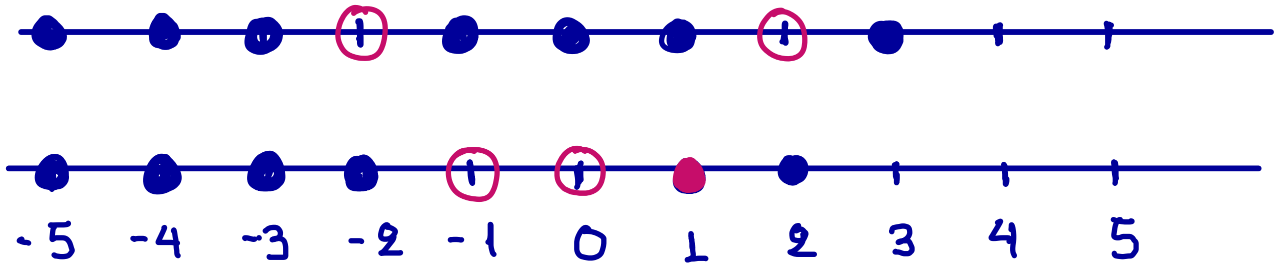

$H(\lambda) = \{3, 2, 4, 0, 1-(-1), 1-0, 1-(-2), 1-2$

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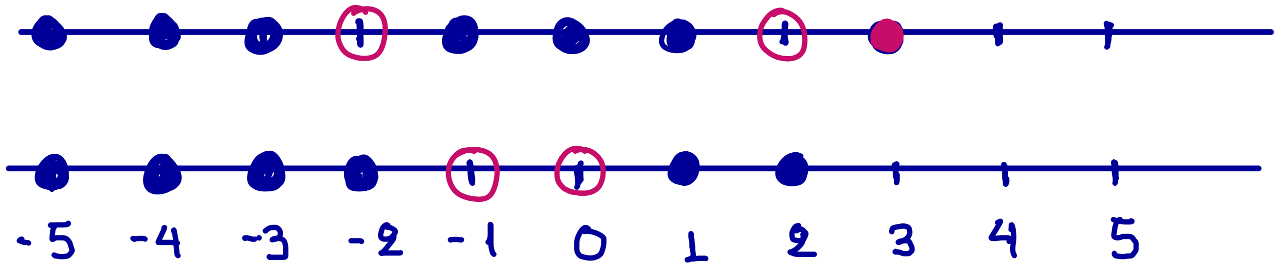

$H(\lambda) = \{3, 2, 4, 0, 2, 1, 3, -1, 3-(-1), 3-0, 3-(-2), 3-2$

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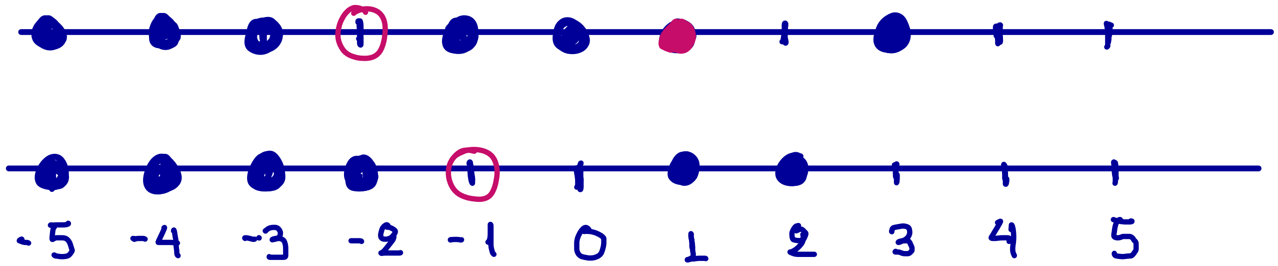

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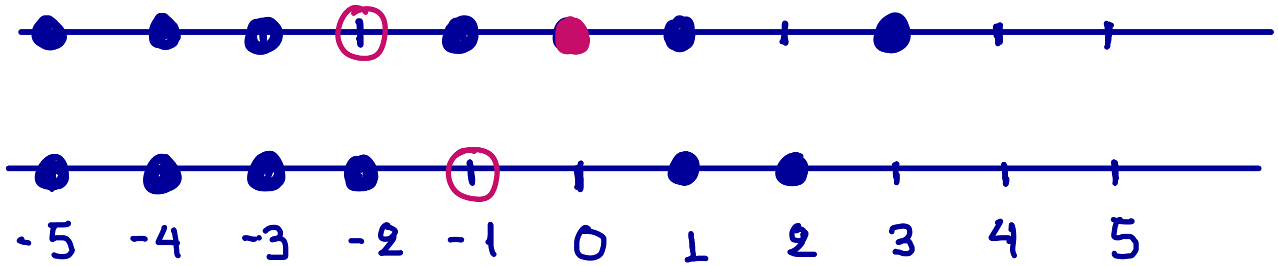

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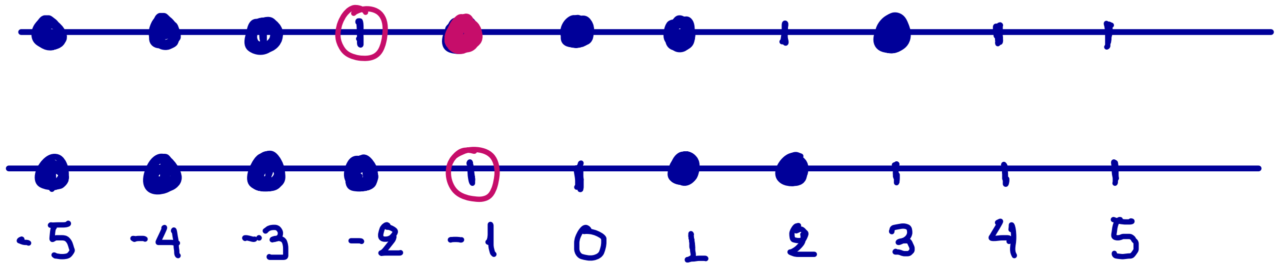

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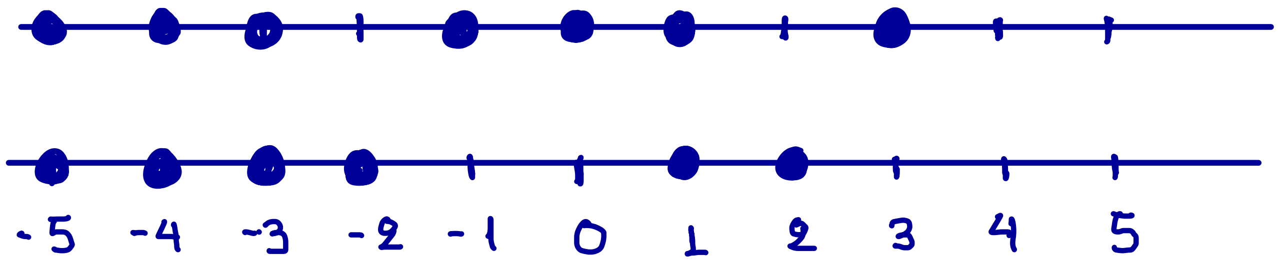

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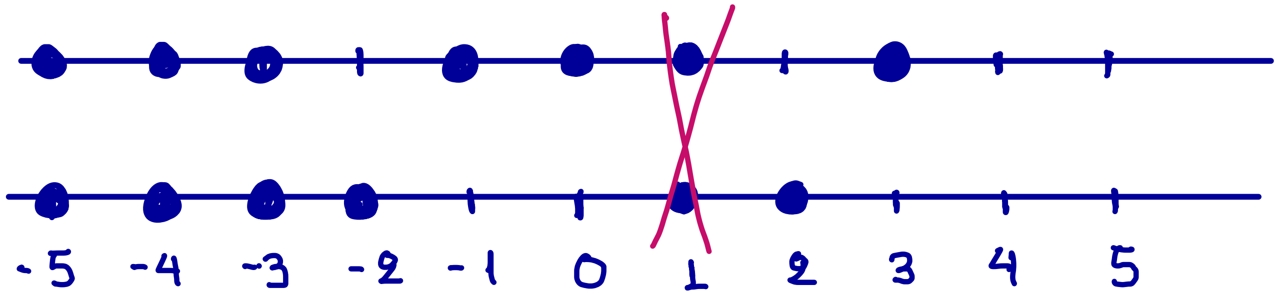

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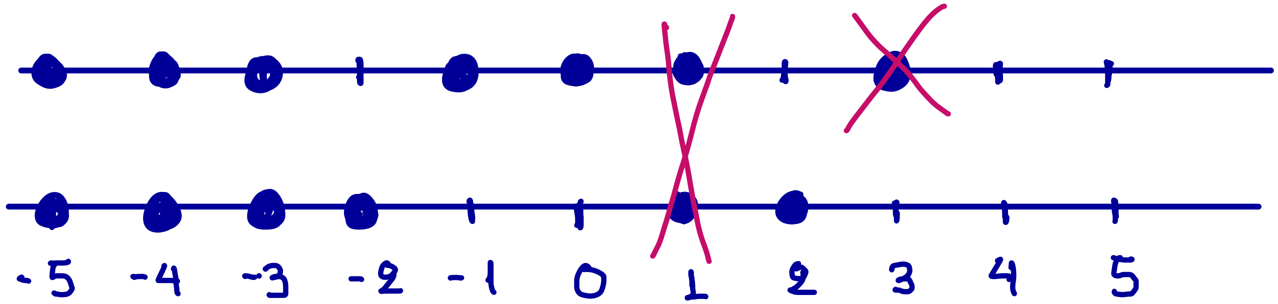

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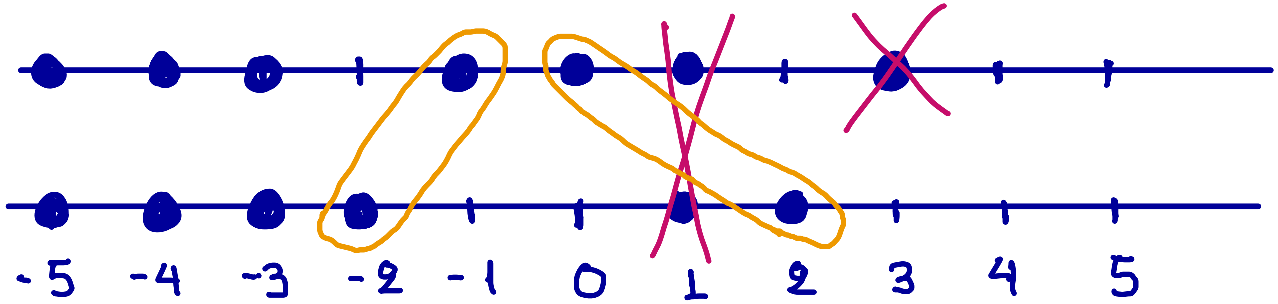

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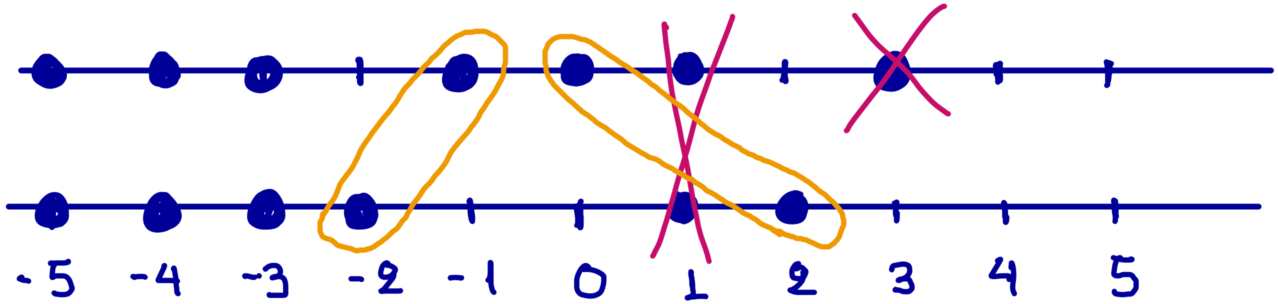

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Conjecture (C.-Jacon)

The defect is a block invariant for cyclotomic Hecke algebras of complex reflection groups.

Happy Birthday Meinolf!