Defects & Weights of Blocks

Maria Chlouveraki
(University of Athens)

j.w. N. Jacon

The β-numbers & the abacus

$\lambda \in \mathcal{P}(n)$ $\quad \lambda = \lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \dots$ $\quad s \in \mathbb{Z}$

$\forall\, j \in \mathbb{Z}_{>0}$, we set $\beta_j := \lambda_j - j + s + 1$

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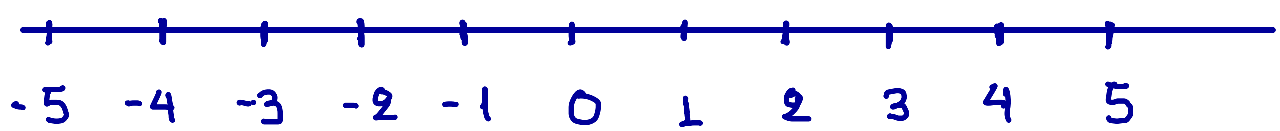
Examples:

$\lambda = (2,2)$
$s = 0$



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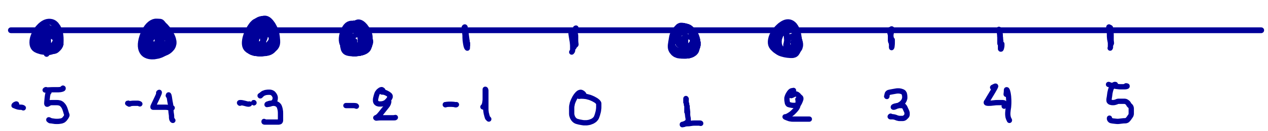

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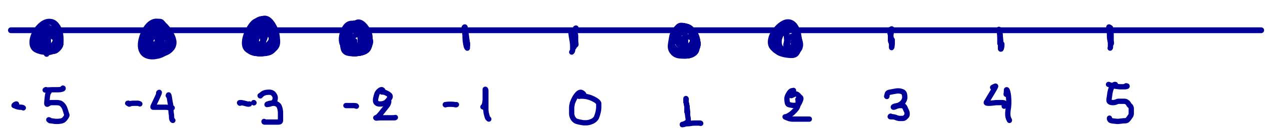

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$s =$ position of the last bead if we slide them all to the left

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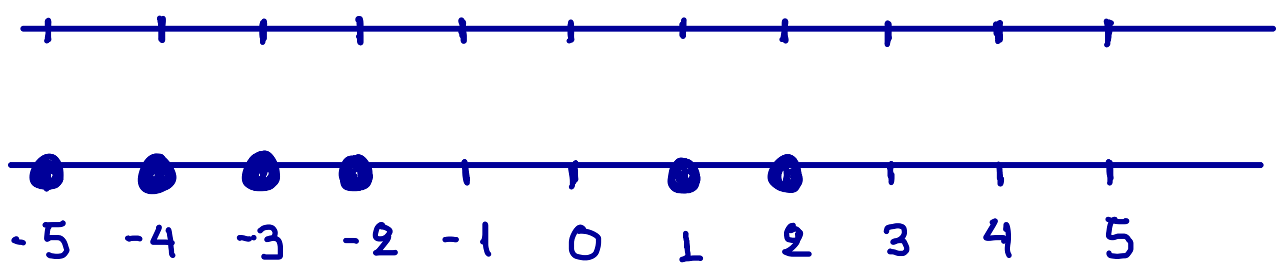

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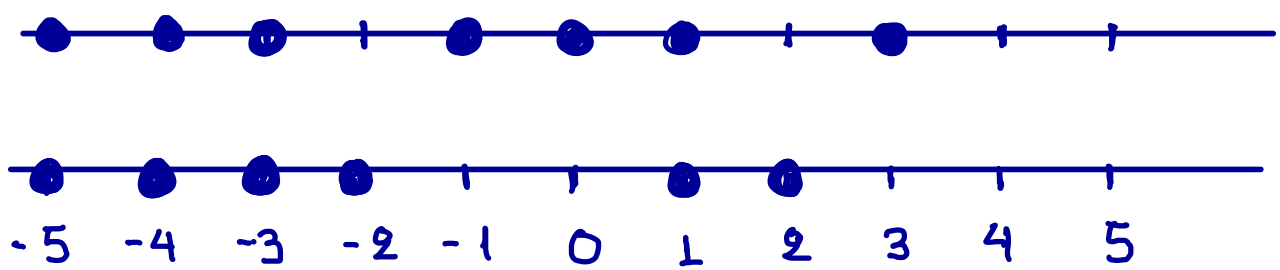

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The generalised hook lengths with a charge

$\lambda^2 = (2,1,1,1)$

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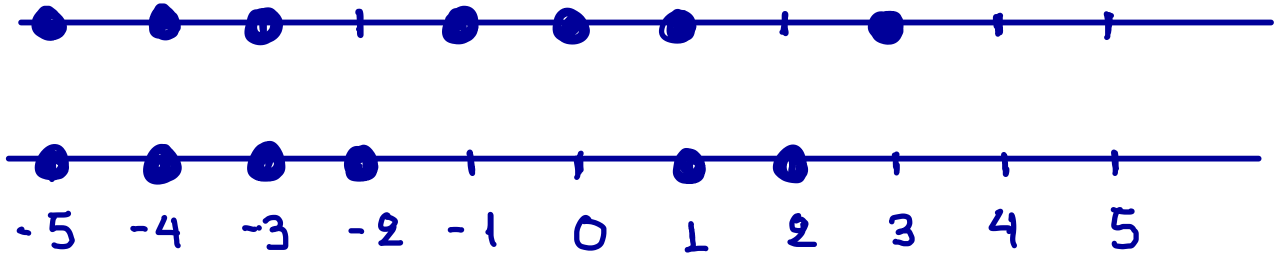

$$\lambda = \big((2,2),(2,1,1,1)\big) \in \mathcal{P}_2(9) \qquad \underline{s} = (0,1)$$

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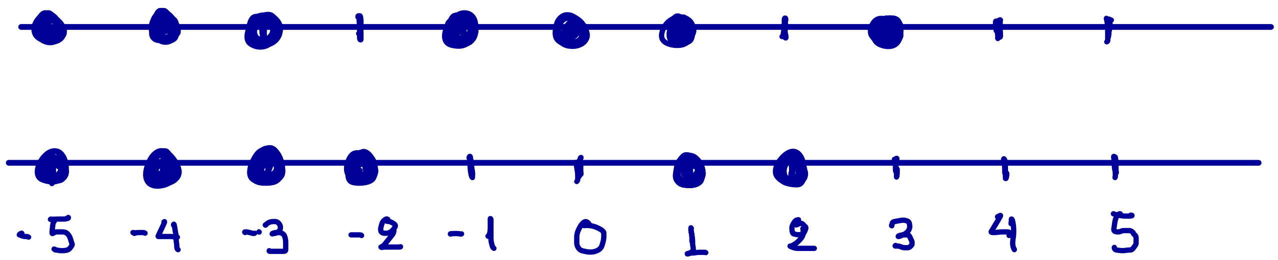

$\lambda = ((2,2),(2,1,1,1)) \in \mathcal{P}_2(9)$ $\underline{s} = (0,1)$

$h^{a,b}_{i,j}(\underline{s}) = \lambda^a_i - i + \lambda^{b\prime}_j - j + 1 + s_a - s_b$ where $(i,j) \in [\lambda^a]$

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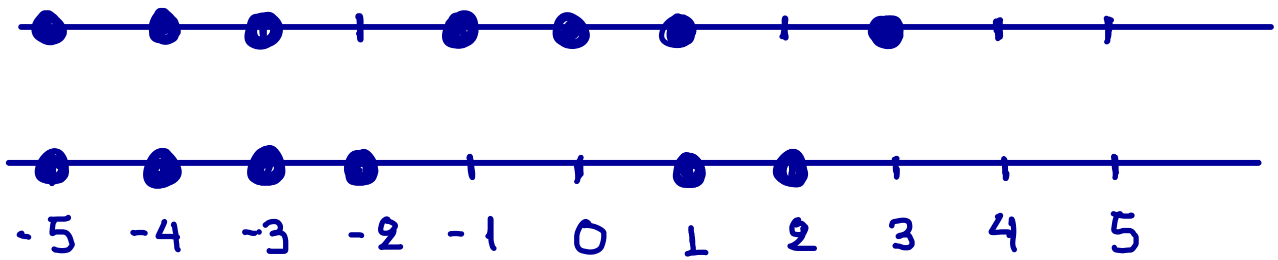

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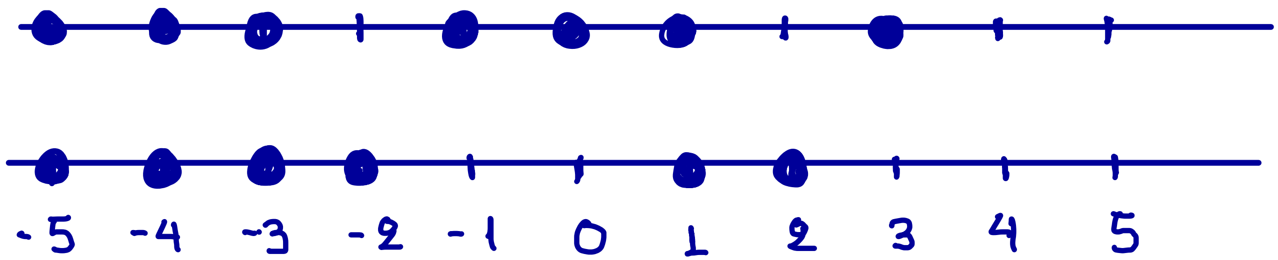

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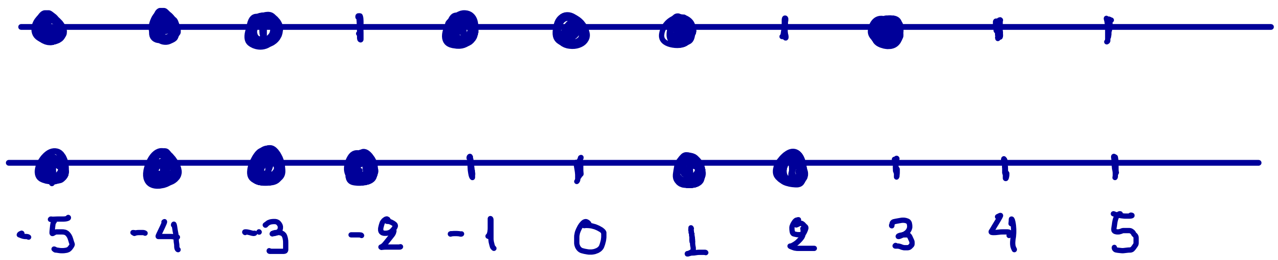

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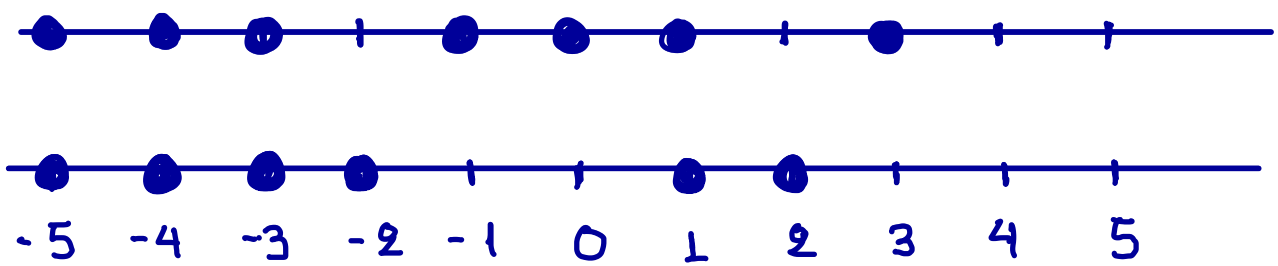

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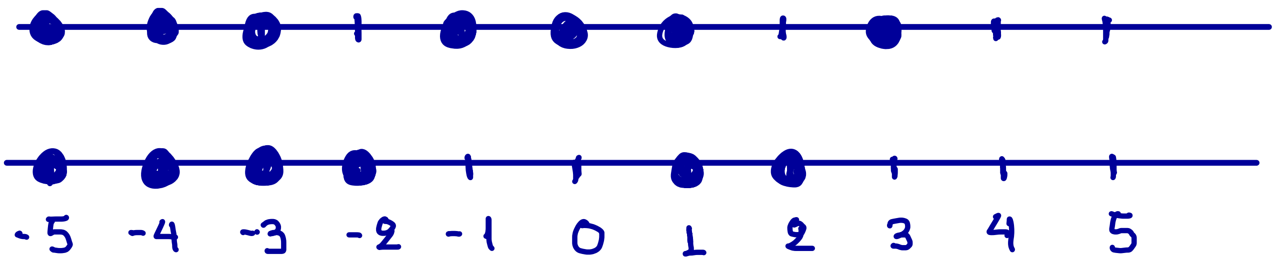

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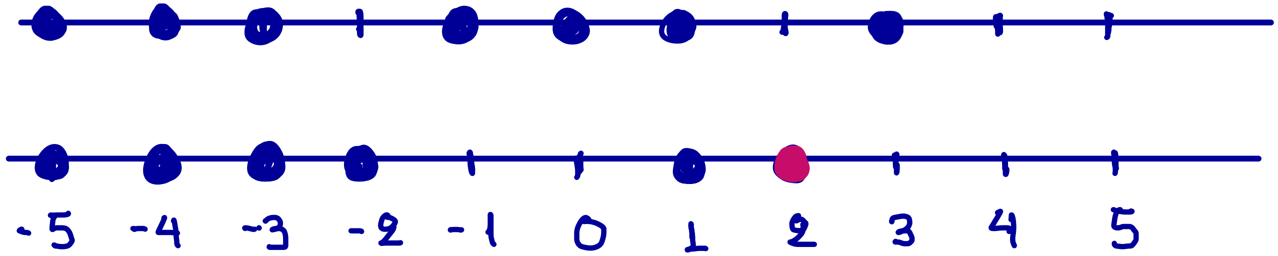

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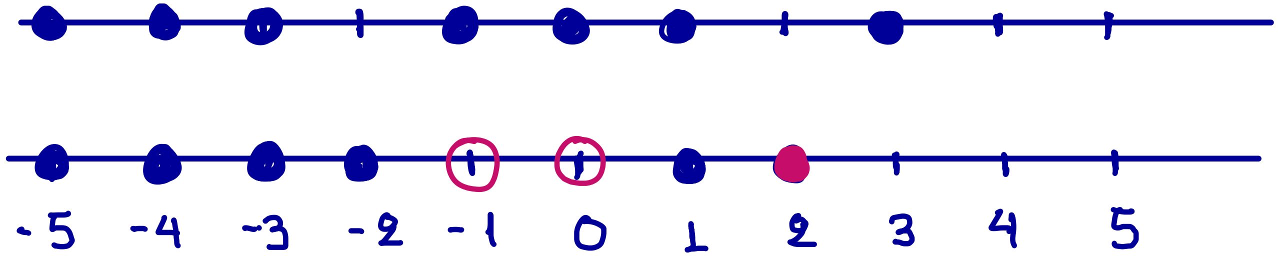

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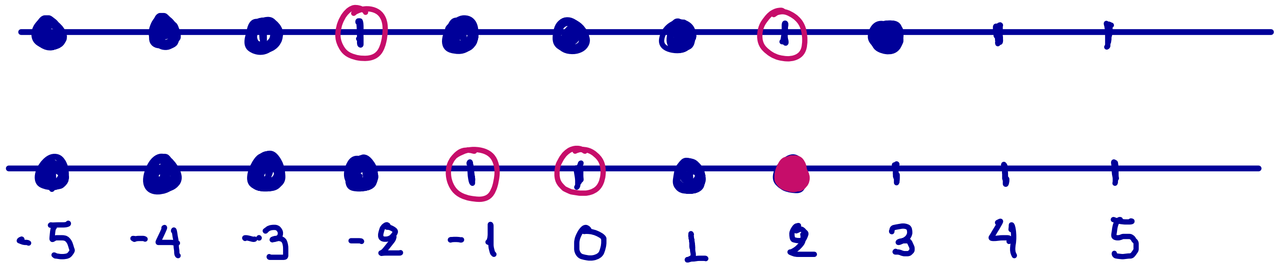

$H(\lambda) = \{ \ 2-(-1), \ 2-0, \ 2-(-2), \ 2-2$

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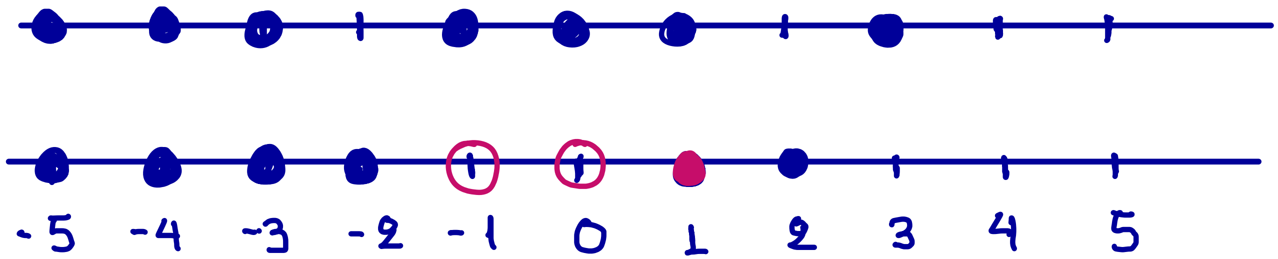

$H(\lambda) = \{3, 2, 4, 0,$

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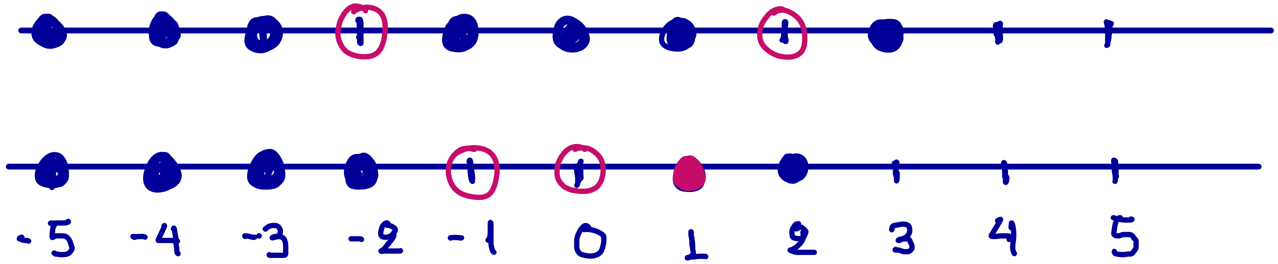

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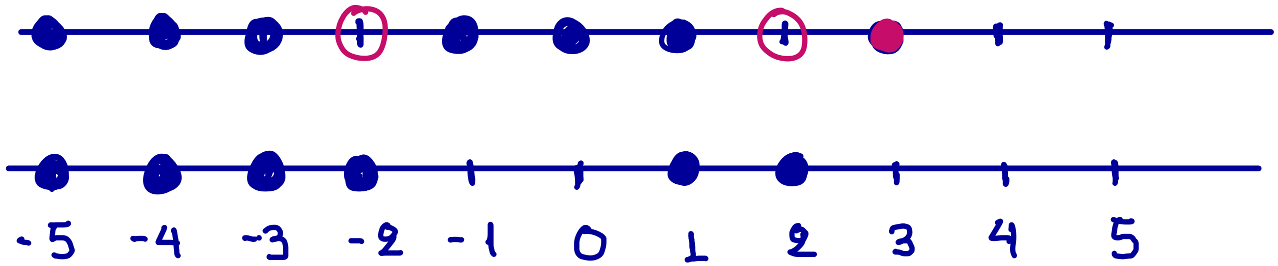

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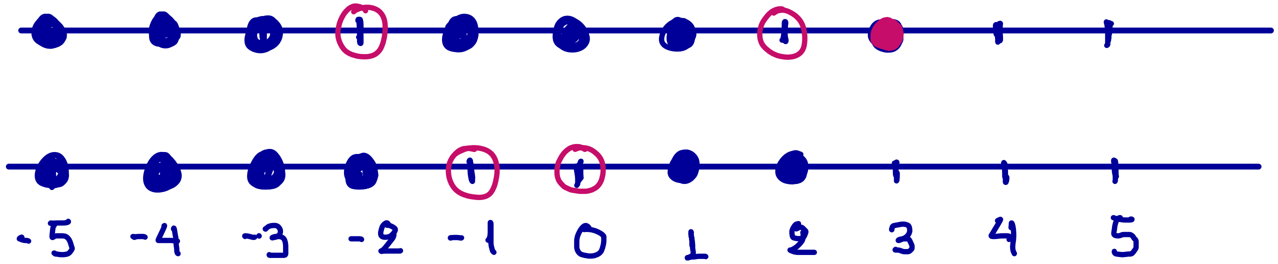

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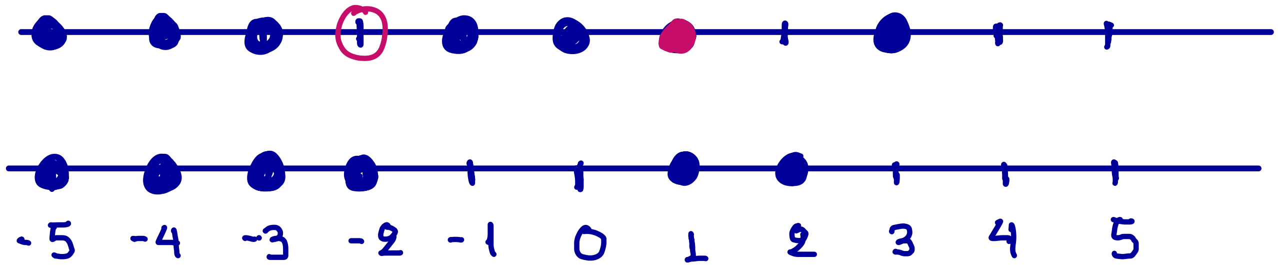

$H(\lambda) = \{3, 2, 4, 0, 2, 1, 3, -1, 4, 3, 5, 1,$

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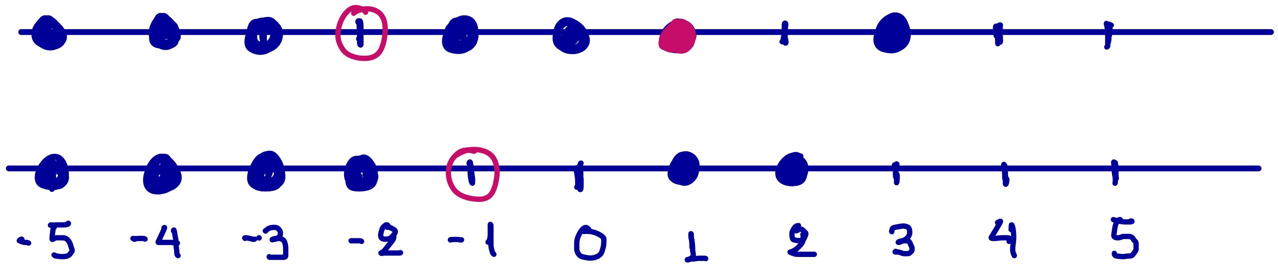

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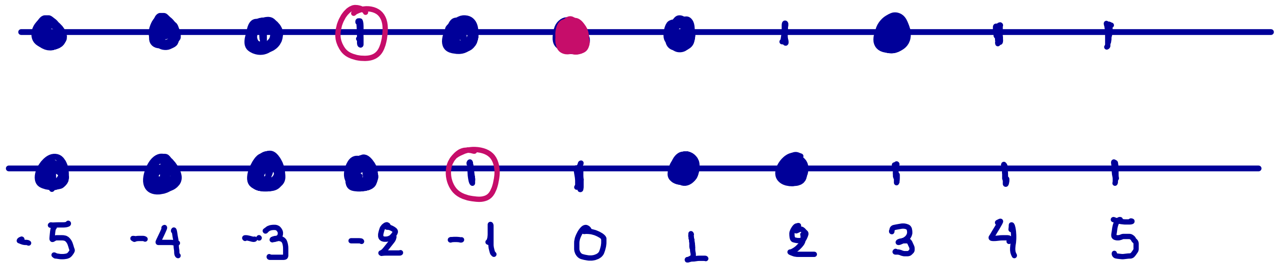

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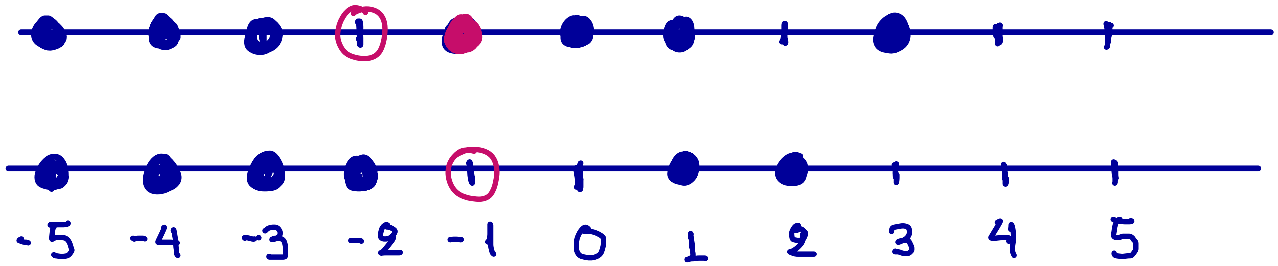

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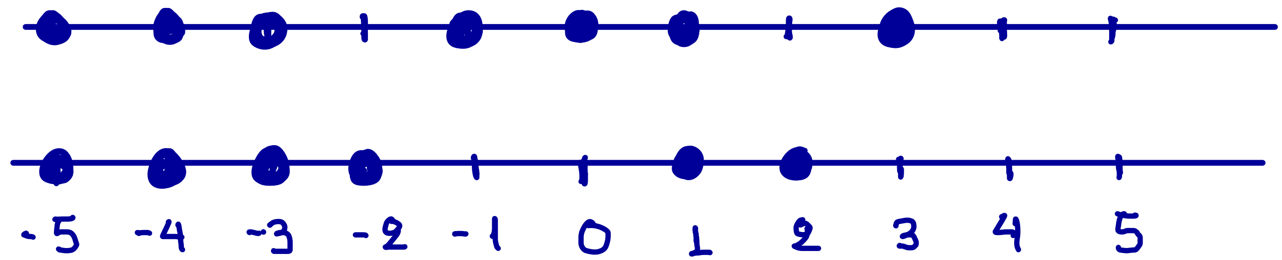

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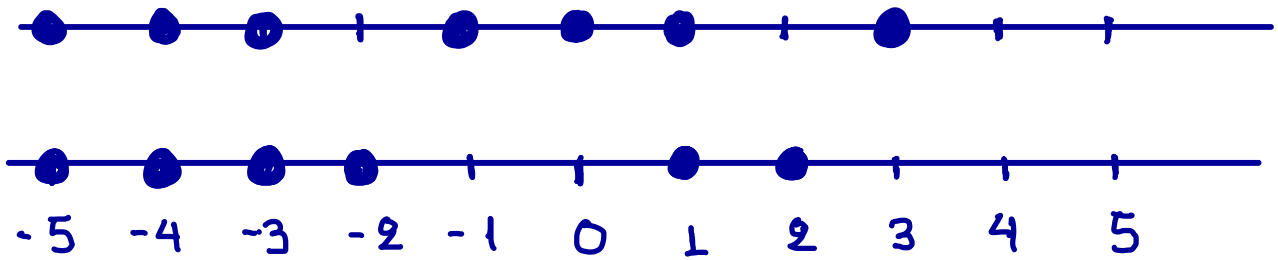

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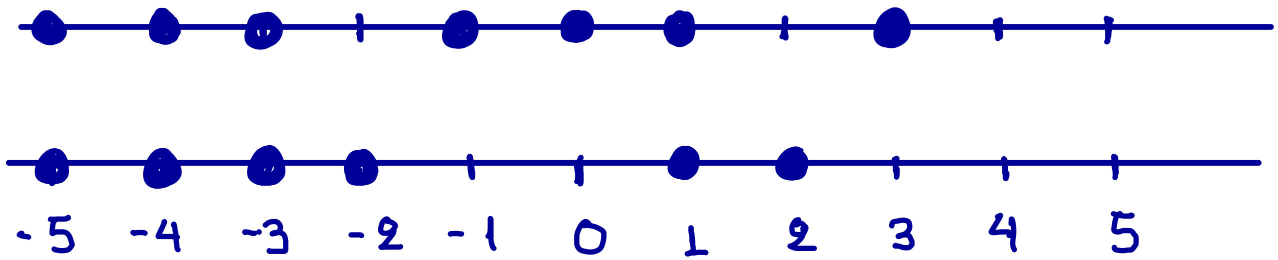

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$$\{-1,0,0,1,1,1,1,2,2,2,2,3,3,3,3,4,4,5\}$$

$m \in \mathbb{Z}$

$$\mathcal{N}_m(\lambda) := \#\{h \in H_\lambda \mid h = m\}$$

We wish to determine $\boxed{\sum_{k \in \mathbb{Z}} \mathcal{N}_{ke}(\lambda) = \operatorname{defect}(\lambda)}$ for $e \geq 2$

We start with $\mathcal{N}_0(\lambda)$

The charged hook lengths equal to 0

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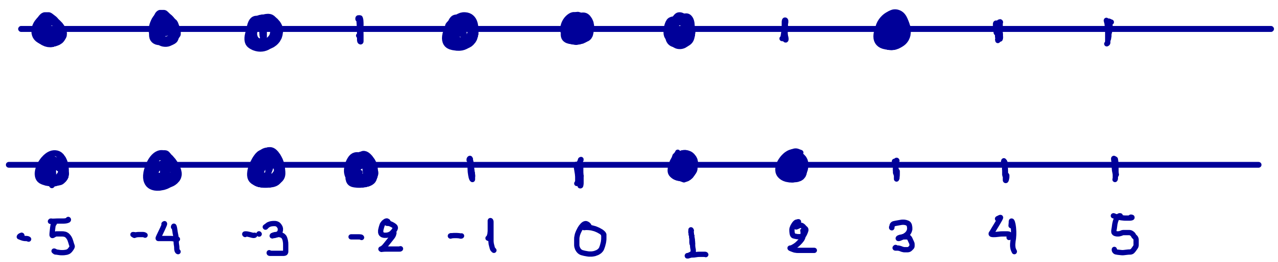

$H(\lambda) = \{-1, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5\}$

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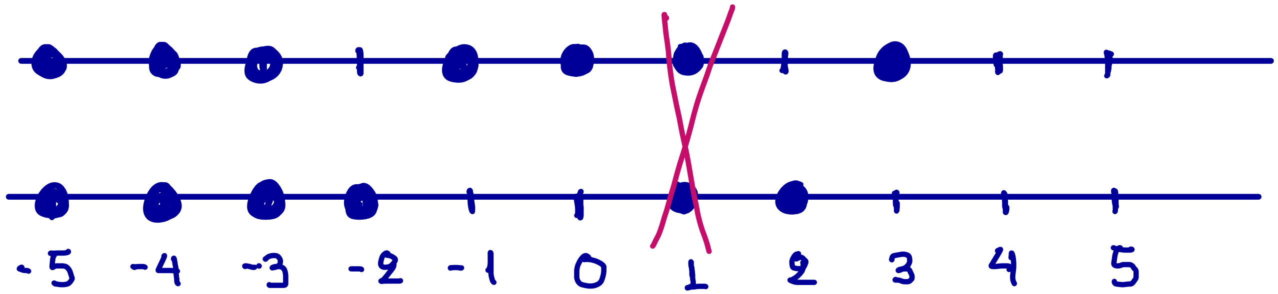

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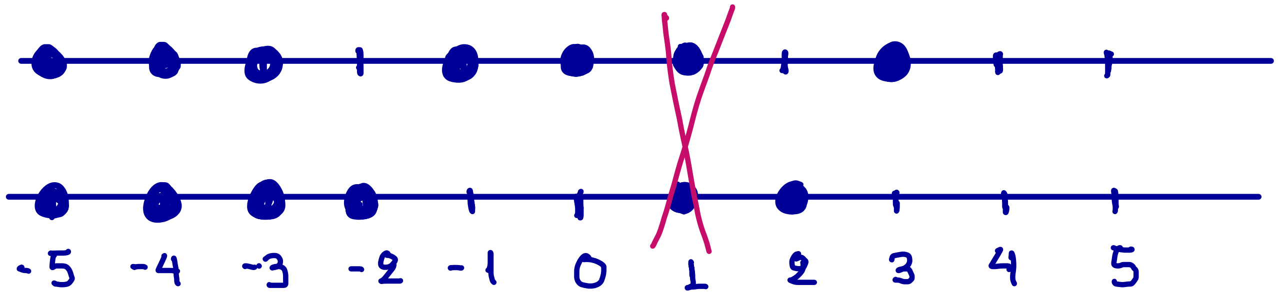

$$H(\lambda) = \{-1,0,0,1,1,1,1,2,2,2,2,3,3,3,3,4,4,5\}$$

$$s_2 - s_1 = 1$$

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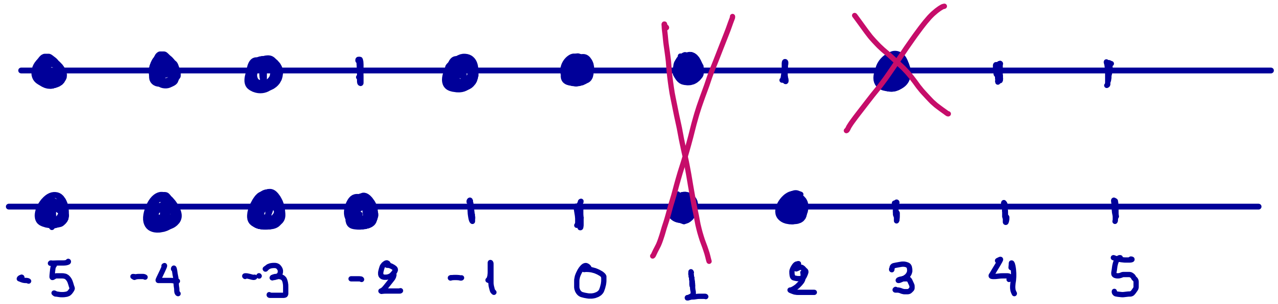

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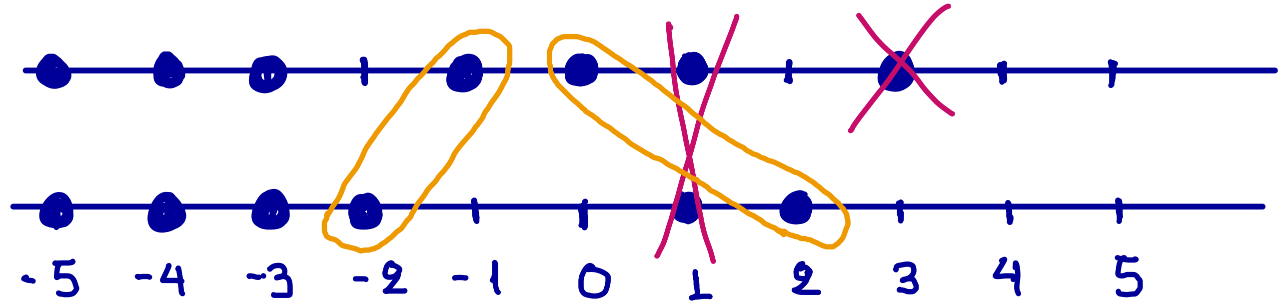

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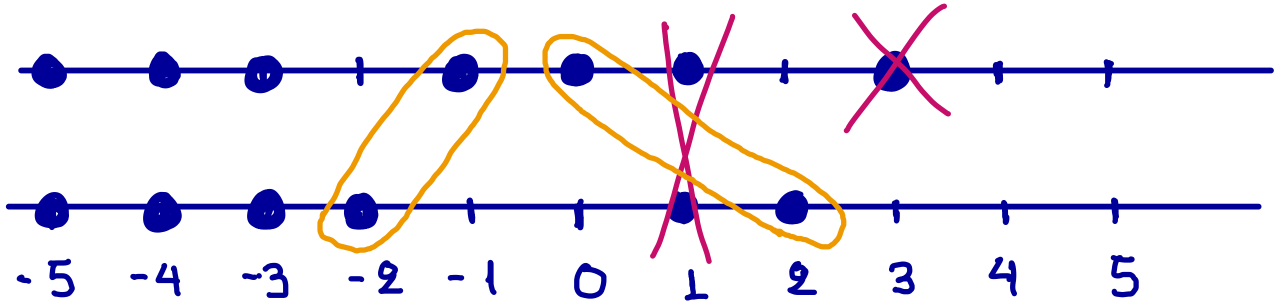

$$H(\lambda) = \{-1, \underline{\underline{0, 0}}, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 5\}$$

$s_2 - s_1 = 1$

The charged hook lengths equal to 0

$\lambda^2 = (2,1,1,1)$

$s_2 = 1$

$\lambda^1 = (2,2)$

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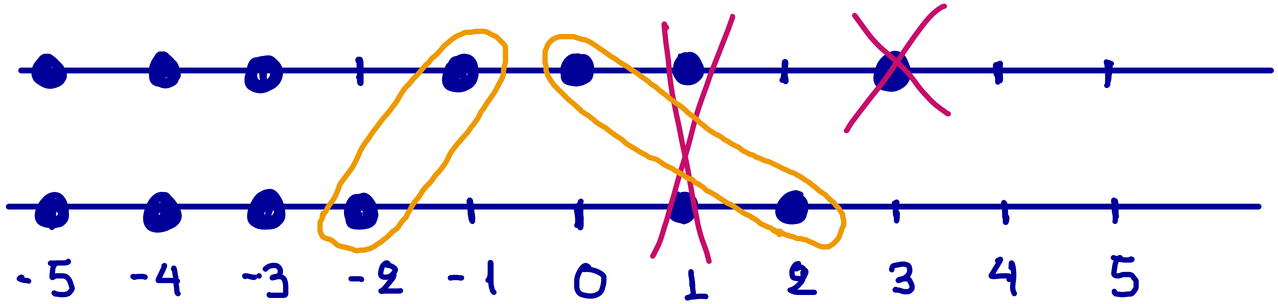

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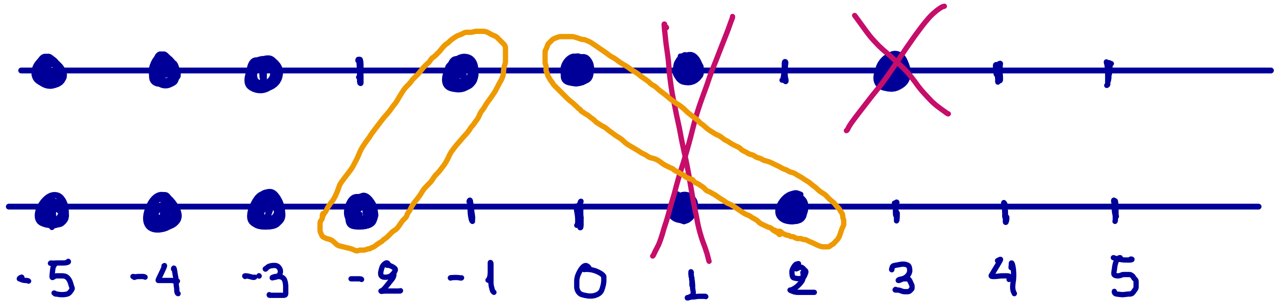

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In general, if we take $s_1 \leq s_2 \leq s_3 \leq \cdots \leq s_r$

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The charged hook lengths congruent to 0 mod e

Let $i \in \{1, \dots, r\}$ and $\beta \in B_i$. For $k \in \mathbb{Z}_{\geq 0}$, we set

$$\mathcal{N}_k(\beta) := \begin{cases} \#\{ j \in \{i+1, \dots, r\} \mid \beta \notin B_j \} & \text{if } k=0 \\ \#\{ j \in \{1, \dots, r\} \mid \beta - ke \notin B_j \} & \text{if } k>0 \end{cases}$$

<u>Proposition</u> [C.-Jacon] Take $s_1 \leq s_2 \leq \dots \leq s_r \leq s_1 + e$

- $\mathcal{N}_0(\lambda) = \sum_{1 \leq i \leq r} \sum_{\beta \in B_i} \mathcal{N}_0(\beta)$
- $\mathcal{N}_{ke}(\lambda) + \mathcal{N}_{-ke}(\lambda) = \sum_{1 \leq i \leq r} \sum_{\beta \in B_i} \mathcal{N}_k(\beta)$ for $k \in \mathbb{Z}_{>0}$
- $\mathrm{Defect}(\lambda) = \sum_{1 \leq i \leq r} \sum_{\beta \in B_i} \sum_{k \geq 0} \mathcal{N}_k(\beta)$

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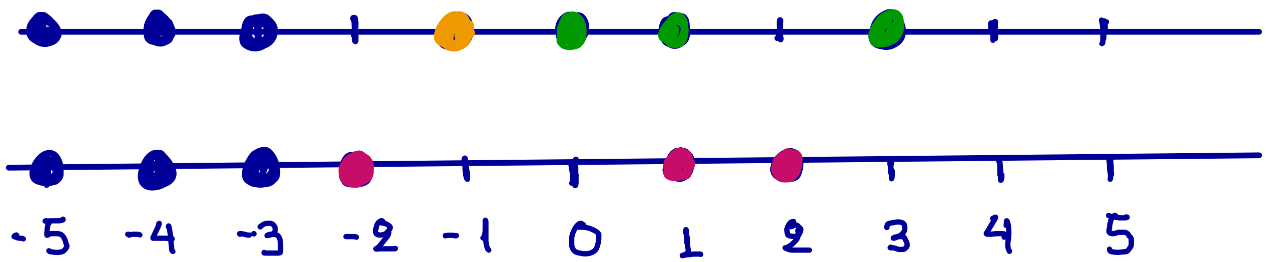

$\boxed{e = 2}$

$$\mathcal{N}_k(\beta) := \begin{cases} \#\{ j \in \{i+1,\dots,r\} \mid \beta \notin B_j \} & \text{if } k = 0 \\ \#\{ j \in \{1,\dots,r\} \mid \beta - 2k \notin B_j \} & \text{if } k > 0 \end{cases}$$

$\mathcal{N}_0(-2) = 1$

$\mathcal{N}_0(1) = 0 \qquad \mathcal{N}_1(1) = 1$

$\mathcal{N}_0(2) = 1 \qquad \mathcal{N}_1(2) = 1 \qquad \mathcal{N}_2(2) = 1$

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