Families of characters of the imprimitive complex reflection groups

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Expansion of Combinatorial Representation Theory

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- A finite reflection group on $\mathbb{C}$ is called a complex reflection group.

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- or W is isomorphic to an exceptional group G_n $(n = 4, \ldots, 37)$.

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- We choose a set of indeterminates $\mathbf{u} = (u_{s,j})_{s,\, 0 \leq j \leq \mathbf{o}(s)-1}$, where s runs over the set of generators of W and $\mathbf{o}(s)$ denotes the order of s (if s and t are conjugate in W, then $u_{s,j} = u_{t,j}$ for all j).

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Theorem (Malle)

Let $\mathbf{v} = (v_{s,j})_{s,j}$ be a set of indeterminates such that, for all s, j, we have

$$v_{s,j}^{|\mu(K)|} := \zeta_{\mathbf{o}(s)}^{-j} u_{s,j},$$

where $\mu(K)$ is the group of all the roots of unity in K. Then the $K(\mathbf{v})$-algebra $K(\mathbf{v})\mathcal{H}(W)$ is split semisimple.

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By "Tits' deformation theorem", the specialization $v_{s,j} \mapsto 1$ induces a bijection

$$\begin{array}{ccc} \mathrm{Irr}(K(\mathbf{v})\mathcal{H}(W)) & \leftrightarrow & \mathrm{Irr}(W) \\ \chi_{\mathbf{v}} & \mapsto & \chi \end{array}$$

Generic Schur elements

The generic Hecke algebra is endowed with a canonical symmetrizing form t. We have that

$$t = \sum_{\chi \in \mathrm{Irr}(W)} \frac{1}{s_\chi} \chi_{\mathbf{v}},$$

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- Ψ is a K-cyclotomic polynomial in one variable,
- M is a primitive monomial of degree 0, *i.e.*, if $M = \prod_{s,j} v_{s,j}^{a_{s,j}}$, then $\gcd(a_{s,j}) = 1$ and $\sum_{s,j} a_{s,j} = 0$.

Schur elements of G_2 : $X_0^2 := u_0$, $X_1^2 := -u_1$, $Y_0^2 := w_0$, $Y_1^2 := -w_1$.

$$s_1 = \Phi_4(X_0X_1^{-1}) \cdot \Phi_4(Y_0Y_1^{-1}) \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_0X_1^{-1}Y_1^{-1})$$

$$s_2 = 2 \cdot X_1^2X_0^{-2} \cdot \Phi_3(X_0Y_0X_1^{-1}Y_1^{-1}) \cdot \Phi_6(X_0Y_1X_1^{-1}Y_0^{-1})$$

$$\Phi_4(x) = x^2 + 1, \quad \Phi_3(x) = x^2 + x + 1, \quad \Phi_6(x) = x^2 - x + 1.$$

Cyclotomic Hecke algebras

Let y be an indeterminate. A cyclotomic specialization of $\mathcal{H}$ is a $\mathbb{Z}_K$-algebra morphism $\phi : \mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}] \to \mathbb{Z}_K[y, y^{-1}]$ of the form:

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Proposition (C.)

The algebra $K(y)\mathcal{H}_\phi$ is split semisimple.

By "Tits' deformation theorem", we obtain that the specialization $v_{s,j} \mapsto 1$ induces the following bijections :

$$\begin{array}{ccccc} \mathrm{Irr}(K(\mathbf{v})\mathcal{H}) & \leftrightarrow & \mathrm{Irr}(K(y)\mathcal{H}_\phi) & \leftrightarrow & \mathrm{Irr}(W) \\ \chi_{\mathbf{v}} & \mapsto & \chi_\phi & \mapsto & \chi \end{array}$$

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Proposition

The Schur element $s_{\chi_\phi}(y)$ associated to the irreducible character χ_ϕ of $K(y)\mathcal{H}_\phi$ is a Laurent polynomial in y of the form

$$s_{\chi_\phi}(y) = \psi_{\chi_\phi} y^{a_{\chi_\phi}} \prod_{\Phi \in C_K} \Phi(y)^{n_{\chi_\phi,\Phi}},$$

where $\psi_{\chi_\phi} \in \mathbb{Z}_K$, $a_{\chi_\phi} \in \mathbb{Z}$, $n_{\chi_\phi,\Phi} \in \mathbb{N}$ and C_K is a set of K-cyclotomic polynomials.

Rouquier blocks of $\mathcal{H}_\phi$

The Rouquier blocks of the cyclotomic Hecke algebra $\mathcal{H}_\phi$ are the blocks of the algebra $\mathcal{R}_K(y)\mathcal{H}_\phi$, where

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i.e., the partition $\mathcal{RB}(\mathcal{H}_\phi)$ of $\mathrm{Irr}(W)$ minimal for the property:

$$\text{For all } B \in \mathcal{RB}(\mathcal{H}_\phi) \text{ and } h \in \mathcal{H}_\phi, \sum_{\chi \in B} \frac{\chi_\phi(h)}{s_{\chi_\phi}} \in \mathcal{R}_K(y).$$

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W c.r.g. (non-Weyl) : Rouquier blocks $\equiv$ "families of characters"

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A primitive monomial M is called essential for W if it is $\mathfrak{p}$-essential for some prime ideal $\mathfrak{p}$ of $\mathbb{Z}_K$.

Schur elements of G_2 : 2-essential in purple, 3-essential in green.

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- If the integers $n_{s,j}$ belong to no essential hyperplane, then the Rouquier blocks of $\mathcal{H}_\phi$ are called Rouquier blocks associated with no essential hyperplane.

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- If the integers $n_{s,j}$ belong to no essential hyperplane, then the Rouquier blocks of $\mathcal{H}_\phi$ are called Rouquier blocks associated with no essential hyperplane.
- If the integers $n_{s,j}$ belong to exactly one essential hyperplane H, then the Rouquier blocks of $\mathcal{H}_\phi$ are called Rouquier blocks associated with the essential hyperplane H.

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Theorem (C.)

Let $\phi : v_{s,j} \mapsto y^{n_{s,j}}$ be a cyclotomic specialization. The Rouquier blocks of $\mathcal{H}_\phi$ is a partition generated by the Rouquier blocks associated with the essential hyperplanes that the $n_{s,j}$ belong to.

Combinatorics

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The integer $|\lambda| := \lambda_1 + \lambda_2 + \ldots + \lambda_h$ is called the size of λ. We also say that λ is a partition of $|\lambda|$.

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Example: If $\lambda = (4, 2, 2, 1)$, then $\beta_\lambda[3] = (10, 7, 6, 4, 2, 1, 0)$.

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From now on, we suppose that we have a given "weight system", *i.e.*, a family of integers

$$m := (m^{(0)}, m^{(1)}, \ldots, m^{(d-1)}).$$

We define the m-charged height of λ to be the integer

$$hc_\lambda := \max\{hc^{(a)} \,|\, (0 \leq a \leq d-1)\},$$

where

$$hc^{(0)} := h^{(0)} - m^{(0)}, hc^{(1)} := h^{(1)} - m^{(1)}, \ldots, hc^{(d-1)} := h^{(d-1)} - m^{(d-1)}.$$

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Definition (*m*-charged standard symbol and content)

The *m-charged standard symbol* of λ is the family of numbers defined by

$$Bc_\lambda = (Bc_\lambda^{(0)}, Bc_\lambda^{(1)}, \ldots, Bc_\lambda^{(d-1)}),$$

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The m-charged content of λ is the multiset

$$\mathrm{Contc}_\lambda = Bc_\lambda^{(0)} \cup Bc_\lambda^{(1)} \cup \ldots \cup Bc_\lambda^{(d-1)}.$$

Example: Let us take $d = 2$, $\lambda = ((3), (2, 1))$ and $m = (2, -1)$. Then

- $\beta^{(0)} = (3)$,
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Consequently,

$$Bc_\lambda = \left(\begin{array}{ccccc} 7 & 3 & 2 & 1 & 0 \\ 3 & 1 & & & \end{array} \right).$$

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We have $\mathrm{Contc}_\lambda = \{0, 1, 1, 2, 3, 3, 7\}$.

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Ariki-Koike algebras

The "generic" Ariki-Koike algebra associated to $G(d,1,r)$ is the algebra $\mathcal{H}_{d,r}$ generated over the Laurent polynomial ring in $d+1$ indeterminates

$$\mathbb{Z}[u_0, u_0^{-1}, u_1, u_1^{-1}, \ldots, u_{d-1}, u_{d-1}^{-1}, x, x^{-1}]$$

by the elements $\mathbf{s}, \mathbf{t}_1, \mathbf{t}_2, \ldots, \mathbf{t}_{r-1}$ satisfying the relations

- $\mathbf{s}\mathbf{t}_1\mathbf{s}\mathbf{t}_1 = \mathbf{t}_1\mathbf{s}\mathbf{t}_1\mathbf{s}$,
- $\mathbf{s}\mathbf{t}_j = \mathbf{t}_j\mathbf{s}$, for all $j = 2, \ldots, r-1$,
- $\mathbf{t}_{j-1}\mathbf{t}_j\mathbf{t}_{j-1} = \mathbf{t}_j\mathbf{t}_{j-1}\mathbf{t}_j$, for all $j = 2, \ldots, r-1$,
- $\mathbf{t}_i\mathbf{t}_j = \mathbf{t}_j\mathbf{t}_i$, for all $1 \leq i, j \leq r-1$ with $|i-j| > 1$,
- $(\mathbf{s} - u_0)(\mathbf{s} - u_1)\ldots(\mathbf{s} - u_{d-1}) = 0$,
- $(\mathbf{t}_j - x)(\mathbf{t}_j + 1) = 0$, for all $j = 1, \ldots, r-1$.

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Let

$$\phi : \begin{cases} u_j \mapsto \zeta_d^j q^{m_j}, (0 \leq j < d), \\ x \mapsto q^n \end{cases}$$

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Let λ, μ be two d-partitions of r. The characters χ_λ and χ_μ are in the same Rouquier block associated with the essential hyperplane $N = 0$ if and only if

$$|\lambda^{(a)}| = |\mu^{(a)}| \text{ for all } a = 0, 1, \ldots, d-1.$$

Let $H : kN + M_s - M_t = 0$ be an essential hyperplane for $G(d, 1, r)$ and let

$$\phi : \begin{cases} u_j \mapsto \zeta_d^j q^{m_j}, (0 \leq j < d), \\ x \mapsto q^n \end{cases}$$

be a cyclotomic specialization such that $kn + m_s - m_t = 0$ and the integers n and m_j $(0 \leq j < d)$ belong to no other essential hyperplane for $G(d, 1, r)$.

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Theorem (Broué-Kim)

Let λ, μ be two d-partitions of r. If the irreducible characters $(\chi_\lambda)_\phi$ and $(\chi_\mu)_\phi$ are in the same Rouquier block of $(\mathcal{H}_{d,r})_\phi$, then $\mathrm{Contc}_\lambda = \mathrm{Contc}_\mu$ with respect to the weight system $(m_0, m_1, \ldots, m_{d-1})$.

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Let λ^{st} and μ^{st} be as above and set $l := |\lambda^{st}| = |\mu^{st}|$.

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Let λ^{st} and μ^{st} be as above and set $l := |\lambda^{st}| = |\mu^{st}|$. Let us consider the Ariki-Koike algebra $\mathcal{H}_{2,l}$ of $G(2,1,l)$ over the Laurent polynomial ring

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$$\mathbb{Z}[U_0, U_0^{-1}, U_1, U_1^{-1}, X, X^{-1}]$$

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By the theorem of Broué-Kim, we have $\mathrm{Contc}_{\lambda^{st}} = \mathrm{Contc}_{\mu^{st}}$ with respect to the weight system (m_s, m_t) if and only if the corresponding characters of $G(2,1,l)$ belong to the same Rouquier block of $(\mathcal{H}_{2,l})_\vartheta$.

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$$\begin{array}{lll}
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\lambda_{(1,1),0} = ((1,1),\emptyset,\emptyset), & \lambda_{(1,1),1} = (\emptyset,(1,1),\emptyset), & \lambda_{(1,1),2} = (\emptyset,\emptyset,(1,1)), \\
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\end{array}$$

Example: Let $W := G(3,1,2)$. The irreducible characters of W are parametrized by the 3-partitions of 2. These are:

$$\begin{array}{lll} \lambda_{(2),0} = ((2),\emptyset,\emptyset), & \lambda_{(2),1} = (\emptyset,(2),\emptyset), & \lambda_{(2),2} = (\emptyset,\emptyset,(2)), \\ \lambda_{(1,1),0} = ((1,1),\emptyset,\emptyset), & \lambda_{(1,1),1} = (\emptyset,(1,1),\emptyset), & \lambda_{(1,1),2} = (\emptyset,\emptyset,(1,1)), \\ \lambda_{\emptyset,0} = (\emptyset,(1),(1)), & \lambda_{\emptyset,1} = ((1),\emptyset,(1)), & \lambda_{\emptyset,2} = ((1),(1),\emptyset). \end{array}$$

The generic Ariki-Koike algebra associated to W is the algebra $\mathcal{H}_{3,2}$ generated over the Laurent polynomial ring in 4 indeterminates

$$\mathbb{Z}[u_0, u_0^{-1}, u_1, u_1^{-1}, u_2, u_2^{-1}, x, x^{-1}]$$

by the elements $\mathbf{s}$ and $\mathbf{t}$ satisfying the relations

- $\mathbf{stst} = \mathbf{tsts}$,
- $(\mathbf{s} - u_0)(\mathbf{s} - u_1)(\mathbf{s} - u_2) = (\mathbf{t} - x)(\mathbf{t} + 1) = 0.$

Let

$$\phi : \begin{cases} u_j \mapsto \zeta_3^j q^{m_j}, (0 \leq j \leq 2), \\ x \mapsto q^n \end{cases}$$

be a cyclotomic specialization for $\mathcal{H}_{3,2}$.

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2. If $\lambda^{01} := (\lambda^{(0)}, \lambda^{(1)})$ and $\mu^{01} := (\mu^{(0)}, \mu^{(1)})$, then $\mathrm{Contc}_{\lambda^{01}} = \mathrm{Contc}_{\mu^{01}}$ with respect to the weight system $(0, 0)$.

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- In the case where $r = 2$ and e is even, explicit calculations had to be made (and there is no combinatorial description of the Rouquier blocks).