Families of characters for cyclotomic Hecke algebras

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If $K = \mathbb{Q}$, then W is a Weyl group.

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Families of characters	$\longrightarrow$	Rouquier blocks

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Every complex reflection group W has a nice "presentation a la Coxeter":

$$\begin{aligned} G_2 &= < s,t \,|\, (st)^3 = (ts)^3,\ s^2 = t^2 = 1 > \\ &= < s,t \,|\, (st)^3 = (ts)^3,\ (s-1)(s+1) = (t-1)(t+1) = 0 > \end{aligned}$$

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The generic Hecke algebra $\mathcal{H}(W)$ has a presentation of the form:

$$\mathcal{H}(G_2) = < \sigma, \tau | (\sigma\tau)^3 = (\tau\sigma)^3, (\sigma - u_0)(\sigma - u_1) = (\tau - u_2)(\tau - u_3) = 0 >$$

and it's defined over the Laurent polynomial ring $\mathbb{Z}[\mathbf{u}, \mathbf{u}^{-1}]$, where $\mathbf{u} = (u_0, u_1, u_2, u_3)$ is a set of indeterminates.

A theorem by G. Malle provides us with a set of indeterminates $\mathbf{v}$ such that the the $K(\mathbf{v})$-algebra $K(\mathbf{v})\mathcal{H}(W)$ is split semisimple:

$$v_0^2 = u_0,\ v_1^2 = -u_1,\ v_2^2 = u_2,\ v_3^2 = -u_3$$

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By "Tits' deformation theorem", the specialization $v_j \mapsto 1$ induces a bijection

$$\begin{array}{ccc} \mathrm{Irr}(K(\mathbf{v})\mathcal{H}(W)) & \leftrightarrow & \mathrm{Irr}(W) \\ \chi_{\mathbf{v}} & \mapsto & \chi \end{array}$$

Generic Schur elements

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The generic Hecke algebra is endowed with a canonical symmetrizing form t. We have that

$$t = \sum_{\chi \in \mathrm{Irr}(W)} \frac{1}{s_\chi} \chi_{\mathbf{v}},$$

where s_χ is the Schur element associated to $\chi_{\mathbf{v}} \in \mathrm{Irr}(K(\mathbf{v})\mathcal{H}(W))$.

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Theorem (C.)

The generic Schur elements are polynomials in $\mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}]$ whose irreducible factors are of the form $\Psi(M)$, where

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The primitive monomials appearing in the factorization of s_χ are unique up to inversion.

Schur elements of G_2

$$s_1 = \Phi_4(v_0 v_1^{-1}) \cdot \Phi_4(v_2 v_3^{-1}) \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2 v_3^{-1})$$

$$s_2 = 2 \cdot v_1^2 v_0^{-2} \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2^{-1} v_3)$$

$\Phi_4(x) = x^2 + 1$, $\Phi_3(x) = x^2 + x + 1$, $\Phi_6(x) = x^2 - x + 1$.

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Definition

Let y be an indeterminate. A cyclotomic specialization of $\mathcal{H}(W)$ is a $\mathbb{Z}_K$-algebra morphism $\phi : \mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}] \to \mathbb{Z}_K[y, y^{-1}]$ such that

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Proposition (C.)

The algebra $K(y)\mathcal{H}_\phi$ is split semisimple.

By "Tits' deformation theorem", we obtain

$$\begin{array}{ccccc} \mathrm{Irr}(K(\mathbf{v})\mathcal{H}(W)) & \leftrightarrow & \mathrm{Irr}(K(y)\mathcal{H}_\phi) & \leftrightarrow & \mathrm{Irr}(W) \\ \chi_{\mathbf{v}} & \mapsto & \chi_\phi & \mapsto & \chi \end{array}$$

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Proposition

The Schur element $s_{\chi_\phi}(y)$ associated to the irreducible character χ_ϕ of $K(y)\mathcal{H}_\phi$ is a Laurent polynomial in y of the form

$$s_{\chi_\phi}(y) = \psi_{\chi_\phi} y^{a_{\chi_\phi}} \prod_{\Phi \in C_K} \Phi(y)^{n_{\chi_\phi,\Phi}},$$

where $\psi_{\chi_\phi} \in \mathbb{Z}_K$, $a_{\chi_\phi} \in \mathbb{Z}$, $n_{\chi_\phi,\Phi} \in \mathbb{N}$ and C_K is a set of K-cyclotomic polynomials.

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We call Rouquier ring of K the $\mathbb{Z}_K$-subalgebra of $K(y)$

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Definition

The Rouquier blocks of the cyclotomic Hecke algebra $\mathcal{H}_\phi$ are the blocks of $\mathcal{R}_K(y)\mathcal{H}_\phi$, *i.e.*, the partition $\mathcal{BR}(\mathcal{H}_\phi)$ of $\mathrm{Irr}(W)$ minimal for the property:

$$\text{For all } B \in \mathcal{BR}(\mathcal{H}_\phi) \text{ and } h \in \mathcal{H}_\phi, \sum_{\chi \in B} \frac{\chi_\phi(h)}{s_{\chi_\phi}} \in \mathcal{R}_K(y).$$

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We denote by $\mathcal{B}_{\mathfrak{p}}(\mathcal{H}_\phi)$ the partition of $\mathrm{Irr}(W)$ into $\mathfrak{p}$-blocks of $\mathcal{H}_\phi$ (*i.e.*, the blocks of the algebra $\mathbb{Z}_K[y, y^{-1}]_{\mathfrak{p}}\mathcal{H}_\phi$).

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Proposition

The Rouquier blocks of $\mathcal{H}_\phi$ is the partition of $\mathrm{Irr}(W)$ generated by the partitions $\mathcal{B}_{\mathfrak{p}}(\mathcal{H}_\phi)$, where $\mathfrak{p}$ runs over the set of prime ideals of $\mathbb{Z}_K$.

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Definition

A primitive monomial M in $\mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}]$ is called $\mathfrak{p}$-essential for W if there exists an irreducible character χ of W and a K-cyclotomic polynomial Ψ such that

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$\mathfrak{p}$-essential monomials and $\mathfrak{p}$-blocks

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Set $A := \mathbb{Z}_K[\mathbf{v}, \mathbf{v}^{-1}]$ and let $\mathcal{B}_\mathfrak{p}(\mathcal{H})$ be the partition of $\mathrm{Irr}(W)$ into $\mathfrak{p}$-blocks of $\mathcal{H}(W)$ (*i.e.*, the blocks of the algebra $A_\mathfrak{p}\mathcal{H}(W)$).

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For every $\mathfrak{p}$-essential monomial M for W, there exists a unique partition $\mathcal{B}_{\mathfrak{p}}^M(\mathcal{H})$ of $\mathrm{Irr}(W)$ with the following properties:

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Moreover, the partition $\mathcal{B}_\mathfrak{p}^M(\mathcal{H})$ coincides with the blocks of the algebra $A_{\mathfrak{q}_M}\mathcal{H}(W)$, where $\mathfrak{q}_M := (M-1)A + \mathfrak{p}A$.

The example of G_2

We denote the characters of G_2 as follows:

$$\chi_{1,0},\ \chi_{1,6},\ \chi_{1,3'},\ \chi_{1,3''},\ \chi_{2,1},\ \chi_{2,2}.$$

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Schur elements: 2-essential in purple, 3-essential in green

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$\Phi_4(1) = 2$, $\Phi_3(1) = 3$, $\Phi_6(1) = 1$

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Monomial	$\mathcal{B}_2^M(\mathcal{H})$	$\mathcal{B}_3^M(\mathcal{H})$
1	$(\chi_{2,1}, \chi_{2,2})$	-
M_1	$(\chi_{1,0}, \chi_{1,3'})$, $(\chi_{2,1}, \chi_{2,2})$, $(\chi_{1,6}, \chi_{1,3''})$	-
M_2	$(\chi_{1,0}, \chi_{1,3''})$, $(\chi_{2,1}, \chi_{2,2})$, $(\chi_{1,6}, \chi_{1,3'})$	-
M_3	$(\chi_{2,1}, \chi_{2,2})$	$(\chi_{1,0}, \chi_{1,6}, \chi_{2,2})$
M_4	$(\chi_{2,1}, \chi_{2,2})$	$(\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1})$

Determination of the Rouquier blocks of a cyclotomic Hecke algebra

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$$\phi^s : \quad \begin{array}{ll} v_0 \mapsto y & v_2 \mapsto y \\ v_1 \mapsto 1 & v_3 \mapsto 1 \end{array}$$

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