Families of characters for cyclotomic Hecke algebras

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30 MAIOY 2008
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If $K = \mathbb{Q}$, then $W$ is a **Weyl group**.
Weyl groups
Weyl groups $\rightarrow$ Complex reflection groups
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Finite reductive groups \quad \rightarrow \quad \text{Rouquier blocks}
Weyl groups $\rightarrow$ Complex reflection groups

Finite reductive groups $\rightarrow$ “Spetses” (?)
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Families of characters $\rightarrow$
Weyl groups → Complex reflection groups
Finite reductive groups → “Spetses” (?)
Families of characters → Rouquier blocks
Hecke algebras of complex reflection groups

Every complex reflection group \( W \) has a nice "presentation a la Coxeter":

\[
G_2 = \langle s, t \mid (st)^3 = (ts)^3, s^2 = t^2 = 1 \rangle
\]

The generic Hecke algebra \( H(W) \) has a presentation of the form:

\[
H(G_2) = \langle \sigma, \tau \mid (\sigma \tau)^3 = (\tau \sigma)^3, (\sigma - u_0)(\sigma - u_1) = (\tau - u_2)(\tau - u_3) = 0 \rangle
\]

and it's defined over the Laurent polynomial ring \( \mathbb{Z}[u, u^{-1}] \), where \( u = (u_0, u_1, u_2, u_3) \) is a set of indeterminates.
Hecke algebras of complex reflection groups

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and it’s defined over the Laurent polynomial ring $\mathbb{Z}[u, u^{-1}]$, where $u = (u_0, u_1, u_2, u_3)$ is a set of indeterminates.
A theorem by G. Malle provides us with a set of indeterminates $\mathbf{v}$ such that the $K(\mathbf{v})$-algebra $K(\mathbf{v})\mathcal{H}(\mathcal{W})$ is split semisimple:

\[
\begin{align*}
\nu_0^2 &= u_0, \\
\nu_1^2 &= -u_1, \\
\nu_2^2 &= u_2, \\
\nu_3^2 &= -u_3
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A theorem by G. Malle provides us with a set of indeterminates $v$ such that the $K(v)$-algebra $K(v)\mathcal{H}(W)$ is split semisimple:

$$v_0^2 = u_0, \quad v_1^2 = -u_1, \quad v_2^2 = u_2, \quad v_3^2 = -u_3$$

By "Tits’ deformation theorem", the specialization $v_j \mapsto 1$ induces a bijection

$$\text{Irr}(K(v)\mathcal{H}(W)) \leftrightarrow \text{Irr}(W)$$

$$\chi_v \quad \mapsto \quad \chi$$
The generic Hecke algebra is endowed with a canonical symmetrizing form \( t \). We have that
\[
\sum_{\chi \in \text{Irr}(W)} \chi s_{v_{\chi}} v_{\chi},
\]
where \( s_{\chi} \) is the Schur element associated to \( v_{\chi} \). The irreducible factors are of the form \( \Psi(M) \), where \( \Psi \) is a \( K \)-cyclotomic polynomial in one variable, \( M \) is a primitive monomial of degree 0, i.e., if \( M = \prod v_{a_{j}} \), then \( \gcd(a_{j}) = 1 \) and \( \sum a_{j} = 0 \). The primitive monomials appearing in the factorization of \( s_{\chi} \) are unique up to inversion.
Generic Schur elements

The generic Hecke algebra is endowed with a canonical symmetrizing form \( t \). We have that

\[
\sum_{\chi \in \text{Irr}(W)} \frac{1}{s_{\chi}} \chi_v,
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**Theorem (C.)**

The generic Schur elements are polynomials in $\mathbb{Z}_K[v, v^{-1}]$ whose irreducible factors are of the form $\Psi(M)$, where

- $\Psi$ is a $K$-cyclotomic polynomial in one variable,
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Schur elements of $G_2$

$$s_1 = \Phi_4(v_0 v_1^{-1}) \cdot \Phi_4(v_2 v_3^{-1}) \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2 v_3^{-1})$$

$$s_2 = 2 \cdot v_1^2 v_0^{-2} \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2^{-1} v_3)$$

$\Phi_4(x) = x^2 + 1$, $\Phi_3(x) = x^2 + x + 1$, $\Phi_6(x) = x^2 - x + 1$. 
Cyclotomic Hecke algebras

Definition

Let $y$ be an indeterminate. A cyclotomic specialization of $H(W)$ is a $\mathbb{Z}[K]\langle v, v^{-1}\rangle$-algebra morphism $\phi: \mathbb{Z}[K]\langle v, v^{-1}\rangle \rightarrow \mathbb{Z}[K]\langle y, y^{-1}\rangle$ such that $\phi: v^{n_j} \mapsto y^{n_j}$, with $n_j \in \mathbb{Z}$ for all $j$.

The corresponding cyclotomic Hecke algebra $H_\phi$ is the $\mathbb{Z}[K]\langle y, y^{-1}\rangle$-algebra obtained as the specialization of $H(W)$ via the morphism $\phi$.

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The algebra $K(y)H_\phi$ is split semisimple.

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Cyclotomic Hecke algebras

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The corresponding **cyclotomic Hecke algebra** $\mathcal{H}_\phi$ is the $\mathbb{Z}_K[y, y^{-1}]$-algebra obtained as the specialization of $\mathcal{H}(W)$ via the morphism $\phi$.

Proposition (C.)

The algebra $K(y)\mathcal{H}_\phi$ is split semisimple.
By “Tits’ deformation theorem”, we obtain

\[
\text{Irr}(K(v)\mathcal{H}(W)) \leftrightarrow \text{Irr}(K(y)\mathcal{H}_\phi) \leftrightarrow \text{Irr}(W)
\]

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\chi_v \leftrightarrow \chi_\phi \leftrightarrow \chi
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\[ \chi_v \mapsto \chi_\phi \mapsto \chi \]

**Proposition**

The Schur element \( s_{\chi_\phi}(y) \) associated to the irreducible character \( \chi_\phi \) of \( K(y)\mathcal{H}_\phi \) is a Laurent polynomial in \( y \) of the form

\[
s_{\chi_\phi}(y) = \psi_{\chi_\phi} y^{a_{\chi_\phi}} \prod_{\Phi \in C_K} \Phi(y)^{n_{\chi_\phi,\Phi}},
\]

where \( \psi_{\chi_\phi} \in \mathbb{Z}_K, a_{\chi_\phi} \in \mathbb{Z}, n_{\chi_\phi,\Phi} \in \mathbb{N} \) and \( C_K \) is a set of \( K \)-cyclotomic polynomials.
Rouquier blocks

We call Rouquier ring of \( K \) the \( \mathbb{Z}[K] \)-subalgebra of \( K[y] \):

\[
R_K := \mathbb{Z}[K][y, y^{-1}, (y^n-1)^{-1}]_{n \geq 1}
\]

**Definition**

The Rouquier blocks of the cyclotomic Hecke algebra \( H_\phi \) are the blocks of \( R_K \), i.e., the partition \( BR(H_\phi) \) of \( \text{Irr}(W) \) minimal for the property:

For all \( B \in BR(H_\phi) \) and \( h \in H_\phi \),

\[
\sum_{\chi \in B} \chi \phi(h) s_{\chi} \in R_K[y].
\]
Rouquier blocks

We call **Rouquier ring** of $K$ the $\mathbb{Z}_K$-subalgebra of $K(y)$

$$\mathcal{R}_K(y) := \mathbb{Z}_K[y, y^{-1}, (y^n - 1)^{-1}_{n \geq 1}]$$
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Rouquier blocks

We call Rouquier ring of $K$ the $\mathbb{Z}_K$-subalgebra of $K(y)$

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**Definition**

The Rouquier blocks of the cyclotomic Hecke algebra $\mathcal{H}_\phi$ are the blocks of $\mathcal{R}_K(y)\mathcal{H}_\phi$, i.e., the partition $BR(\mathcal{H}_\phi)$ of $\text{Irr}(W)$ minimal for the property:

For all $B \in BR(\mathcal{H}_\phi)$ and $h \in \mathcal{H}_\phi$, \( \sum_{\chi \in B} \frac{\chi_\phi(h)}{s_{\chi_\phi}} \in \mathcal{R}_K(y) \).
Let $p$ be a prime ideal of $\mathbb{Z}_K$. 
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We denote by $B_p(\mathcal{H}_\phi)$ the partition of $\text{Irr}(W)$ into $p$-blocks of $\mathcal{H}_\phi$ (i.e., the blocks of the algebra $\mathbb{Z}_K[y, y^{-1}]_p\mathcal{H}_\phi$).
Let \( p \) be a prime ideal of \( \mathbb{Z}_K \).

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**Proposition**

The Rouquier blocks of \( \mathcal{H}_\phi \) is the partition of \( \text{Irr}(W) \) generated by the partitions \( B_p(\mathcal{H}_\phi) \), where \( p \) runs over the set of prime ideals of \( \mathbb{Z}_K \).
p-essential monomials
\textbf{Definition}

A primitive monomial $M$ in $\mathbb{Z}_K[v,v^{-1}]$ is called \textit{p-essential for $W$} if there exists an irreducible character $\chi$ of $W$ and a $K$-cyclotomic polynomial $\Psi$ such that

$$\Psi(M) \text{ divides } s^2 \chi(v) \Psi(1).$$
p-essential monomials

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A primitive monomial $M$ in $\mathbb{Z}_K[v, v^{-1}]$ is called *p-essential for* $W$ if there exists an irreducible character $\chi$ of $W$ and a $K$-cyclotomic polynomial $\Psi$ such that

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1. $\Psi(M)$ divides $s_\chi(v)$
2. $\Psi(1) \in p$. 

**Definition**
$p$-essential monomials and $p$-blocks

$\mathcal{A} := \mathbb{Z} K[v, v-1]$ and let $B_p(\mathcal{H})$ be the partition of $\text{Irr}(\mathcal{W})$ into $p$-blocks of $\mathcal{H}(\mathcal{W})$ (i.e., the blocks of the algebra $A_p \mathcal{H}(\mathcal{W})$).

Theorem (C.)

For every $p$-essential monomial $M$ for $\mathcal{W}$, there exists a unique partition $B_M_p(\mathcal{H})$ of $\text{Irr}(\mathcal{W})$ with the following properties:

1. The parts of $B_M_p(\mathcal{H})$ are unions of the parts of $B_p(\mathcal{H})$.

2. The partition $B_p(\mathcal{H})$ is the partition generated by the partitions $B_p(\mathcal{H})$ et $B_M_p(\mathcal{H})$, where $M$ runs over the set of all $p$-essential monomials which are sent to 1 by $\phi$.

Moreover, the partition $B_M_p(\mathcal{H})$ coincides with the blocks of the algebra $A_{qM} \mathcal{H}(\mathcal{W})$, where $q_{M} := (M-1)A + pA$.
Set $A := \mathbb{Z}_K[v, v^{-1}]$ and let $B_p(H)$ be the partition of $\text{Irr}(W)$ into $p$-blocks of $H(W)$ (i.e., the blocks of the algebra $A_p H(W)$).
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Set \( A := \mathbb{Z}_K[v, v^{-1}] \) and let \( B_p(H) \) be the partition of \( \text{Irr}(W) \) into \( p \)-blocks of \( H(W) \) (i.e., the blocks of the algebra \( A_p H(W) \)).

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For every \( p \)-essential monomial \( M \) for \( W \), there exists a unique partition \( B_p^M(H) \) of \( \text{Irr}(W) \) with the following properties:

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p-essential monomials and p-blocks

Set \( A := \mathbb{Z}_K[v, v^{-1}] \) and let \( B_p(\mathcal{H}) \) be the partition of \( \text{Irr}(W) \) into p-blocks of \( \mathcal{H}(W) \) (i.e., the blocks of the algebra \( A_p \mathcal{H}(W) \)).

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2. The partition \( B_p(\mathcal{H}_\phi) \) is the partition generated by the partitions \( B_p(\mathcal{H}) \) et \( B_p^M(\mathcal{H}) \), where \( M \) runs over the set of all p-essential monomials which are sent to 1 by \( \phi \).
Set $A := \mathbb{Z}_K[v, v^{-1}]$ and let $\mathcal{B}_p(\mathcal{H})$ be the partition of $\text{Irr}(W)$ into $p$-blocks of $\mathcal{H}(W)$ (i.e., the blocks of the algebra $A_p \mathcal{H}(W)$).

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Moreover, the partition $\mathcal{B}_p^M(\mathcal{H})$ coincides with the blocks of the algebra $A_{q_M} \mathcal{H}(W)$, where $q_M := (M - 1)A + pA$. 
The example of $G_2$

We denote the characters of $G_2$ as follows:

$\chi_{1,0}, \chi_{1,6}, \chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}, \chi_{2,2}$. 

$\Phi_4(x) = x^2 + 1$, $\Phi_3(x) = x^2 + x + 1$, $\Phi_6(x) = x^2 - x + 1$, $\Phi_4(1) = 2$, $\Phi_3(1) = 3$, $\Phi_6(1) = 1$. 

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Schur elements: 2-essential in purple, 3-essential in green

$$s_1 = \Phi_4(v_0 v_1^{-1}) \cdot \Phi_4(v_2 v_3^{-1}) \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2 v_3^{-1})$$

$$s_2 = 2 \cdot v_1^2 v_0^{-2} \cdot \Phi_3(v_0 v_1^{-1} v_2 v_3^{-1}) \cdot \Phi_6(v_0 v_1^{-1} v_2^{-1} v_3)$$

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The 3-essential monomials for $G_2$ are:

$$M_3 := v_0 v_1^{-1} v_2 v_3^{-1} \text{ and } M_4 := v_0 v_1^{-1} v_2^{-1} v_3.$$
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$$M_3 := v_0 v_1^{-1} v_2 v_3^{-1} \text{ and } M_4 := v_0 v_1^{-1} v_2^{-1} v_3. $$

<table>
<thead>
<tr>
<th>Monomial</th>
<th>$B^M_2(\mathcal{H})$</th>
<th>$B^M_3(\mathcal{H})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(\chi_{2,1}, \chi_{2,2})$</td>
<td>-</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$(\chi_{1,0}, \chi_{1,3'})$, $(\chi_{2,1}, \chi_{2,2})$, $(\chi_{1,6}, \chi_{1,3''})$</td>
<td>-</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$(\chi_{1,0}, \chi_{1,3''})$, $(\chi_{2,1}, \chi_{2,2})$, $(\chi_{1,6}, \chi_{1,3'})$</td>
<td>-</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$(\chi_{2,1}, \chi_{2,2})$</td>
<td>$(\chi_{1,0}, \chi_{1,6}, \chi_{2,2})$</td>
</tr>
<tr>
<td>$M_4$</td>
<td>$(\chi_{2,1}, \chi_{2,2})$</td>
<td>$(\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1})$</td>
</tr>
</tbody>
</table>
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

The only essential monomial sent to 1 is $M_4$. Thus the Rouquier blocks of $H_s$ are:

$(\chi_1, 0), (\chi_1, 6), (\chi_1, 3'), (\chi_1, 3''), (\chi_2, 1), (\chi_2, 2)$.

Determination of the Rouquier blocks of the group algebra $W$:

All essential monomials are sent to 1. We have:

#2-blocks

$(\chi_1, 0), (\chi_1, 6), (\chi_1, 3'), (\chi_1, 3'')$,

$(\chi_2, 1), (\chi_2, 2)$.

#3-blocks

$(\chi_1, 0), (\chi_1, 6), (\chi_2, 2)$,

$(\chi_1, 3'), (\chi_1, 3''), (\chi_2, 1)$.

#1 Rouquier block

$(\chi_1, 0), (\chi_1, 6), (\chi_1, 3'), (\chi_1, 3''), (\chi_2, 1), (\chi_2, 2)$.
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

\[ \phi^S : \quad v_0 \mapsto y \quad v_2 \mapsto y \]

\[ v_1 \mapsto 1 \quad v_3 \mapsto 1 \]

The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( H_s^\phi \) are:

\[ (\chi_1^1, 0), (\chi_1^1, 6), (\chi_1^1, 3') \]

Determining the Rouquier blocks of the group algebra \( \phi^W \):

\[ v_0 \mapsto 1 \quad v_2 \mapsto 1 \]

\[ v_1 \mapsto 1 \quad v_3 \mapsto 1 \]

All essential monomials are sent to 1. We have:

#2 2-blocks

\[ (\chi_1^1, 0, \chi_1^1, 6, \chi_1^1, 3'), (\chi_1^2, 1, \chi_1^2, 2) \]

#2 3-blocks

\[ (\chi_1^1, 0, \chi_1^1, 6, \chi_2^2, 2), (\chi_1^1, 3', \chi_1^1, 3'', \chi_2^1, 1) \]

#1 Rouquier block

\[ (\chi_1^1, 0, \chi_1^1, 6, \chi_1^1, 3', \chi_1^1, 3'', \chi_2^2, 1, \chi_2^2, 2) \]
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

\[ \phi^s : \begin{align*}
v_0 &\mapsto y \quad v_2 \mapsto y \\
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\end{align*} \]

The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( \mathcal{H}_\phi^s \) are:

- \( \chi_{1,0}, \chi_{1,6}, \chi_{1,3}', \chi_{2,1}, \chi_{2,2} \)
- \( \chi_{1,0}, \chi_{1,6}, \chi_{2,2} \)

Maria Chlouveraki (EPFL) Families of characters for cyclotomic Hecke September 14, 2008 16 / 16
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

\[ \phi^s : \begin{align*} v_0 & \mapsto y \quad v_2 \mapsto y \\ v_1 & \mapsto 1 \quad v_3 \mapsto 1 \end{align*} \]

The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( H^s_\phi \) are:

\[ (\chi_{1,0}), (\chi_{1,6}), (\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}, \chi_{2,2}). \]
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Determination of the Rouquier blocks of the group algebra
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

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Determination of the Rouquier blocks of the group algebra

\[ \phi^W : \begin{align*} \nu_0 & \mapsto 1 \\ \nu_2 & \mapsto 1 \\ \nu_1 & \mapsto 1 \\ \nu_3 & \mapsto 1 \end{align*} \]
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

\[ \phi^s : \quad v_0 \mapsto y \quad v_2 \mapsto y \]
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The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( \mathcal{H}_\phi^s \) are:

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Determination of the Rouquier blocks of the group algebra

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All essential monomials are sent to 1. We have:
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

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Determination of the Rouquier blocks of the group algebra

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\[ \#2 \ 2\text{-blocks} \quad (\chi_{1,0}, \chi_{1,6}, \chi_{1,3'}, \chi_{1,3''}), (\chi_{2,1}, \chi_{2,2}) \]
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

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The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( \mathcal{H}^s_\phi \) are:

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Determination of the Rouquier blocks of the group algebra

\[ \phi^W : v_0 \mapsto 1 \quad v_2 \mapsto 1 \]
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All essential monomials are sent to 1. We have:

\#2 2-blocks \( (\chi_{1,0}, \chi_{1,6}, \chi_{1,3'}, \chi_{1,3''}), (\chi_{2,1}, \chi_{2,2}) \)

\#2 3-blocks \( (\chi_{1,0}, \chi_{1,6}, \chi_{2,2}), (\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}) \)
Determination of the Rouquier blocks of a cyclotomic Hecke algebra

\[ \phi^s : \begin{array}{c@{}c@{}c@{}c}
v_0 & \mapsto & y & v_2 \mapsto y \\
v_1 & \mapsto & 1 & v_3 \mapsto 1 \end{array} \]

The only essential monomial sent to 1 is \( M_4 \). Thus the Rouquier blocks of \( H^s_\phi \) are:

\[ (\chi_{1,0}), (\chi_{1,6}), (\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}, \chi_{2,2}). \]

Determination of the Rouquier blocks of the group algebra

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All essential monomials are sent to 1. We have:

- #2 2-blocks: \( (\chi_{1,0}, \chi_{1,6}, \chi_{1,3'}, \chi_{1,3''}), (\chi_{2,1}, \chi_{2,2}) \)
- #2 3-blocks: \( (\chi_{1,0}, \chi_{1,6}, \chi_{2,2}), (\chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}) \)
- #1 Rouquier block: \( (\chi_{1,0}, \chi_{1,6}, \chi_{1,3'}, \chi_{1,3''}, \chi_{2,1}, \chi_{2,2}) \)